Two-dimensional remote air-pollution monitoring via tomography

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We propose to apply computerized tomography to measure a two-dimensional pollutant-concentration map over an area that may contain several potential sources of pollution. A tunable-laser source at the center of the area generates secondary or virtual light sources around the perimeter of the area that play the role of x rays in conventional computerized tomography of the human body.

The use of tunable-laser sources for remote pollution measurements is now well known.\textsuperscript{1,2} Depth-resolved absorption measurements along a line can be obtained by using atmospheric backscattering and time resolving the return signal. Because of the weak backscattered return signal, high transmitted laser energies and large receiver diameters are required in addition to high-speed detection and signal digitization to obtain depth-resolved measurements over a few kilometers' range to a resolution of 10 m.

We propose to use computerized tomography\textsuperscript{3,4} to generate a two-dimensional map of pollutant concentration. Air-pollution measurements by computerized tomography provide several advantages over the differential-absorption method. These include a significantly lower required transmitted laser energy, increased range, and the ability to monitor an area containing several pollutant sources so numerous as to make it impractical to monitor each of them separately. The measurement is implemented by using a single laser source located at the center of the area to generate a number of secondary light sources around the perimeter that are analogous to x-ray sources in conventional tomography of the human body.

Specifically, we suppose that a tunable-laser source L is mounted in the center of a circle of radius R, as shown in Fig. 1, such that its beam can easily be rotated and directed toward the circumference of the circle. At each of n points equally spaced on the circumference, we mount a cylindrical mirror \(M_i\) \((i = 1 \ldots n)\) so that the collimated incident laser beam is reflected in a fan beam over an angle \(\gamma\) across the circle. The beams from \(M_i\) traverse the circular region A and strike a set of m fixed detectors \(D_j\) \((j = 1 \ldots m)\). The mirrors and detectors lie in a common plane P, which may be parallel to the ground at a sufficient height so that the beams are not obscured by hills, chimneys, or buildings.

In traversing the path from \(M_i\) to \(D_j\), the laser beams are attenuated exponentially so that

\[
P_{ij} = P_{ij}^0 \exp\left[-\int_{L_{ij}} \alpha_A(x,y,\lambda) \, ds\right],
\]

where \(P_{ij}^0\) and \(P_{ij}\) are the received and transmitted powers over the path \(L_{ij}\) and \(\alpha_A(x,y,\lambda) = \alpha_R(x,y) + \alpha_M(x,y) + \alpha_{abs}(x,y,\lambda)\) is the atmosphere extinction coefficient composed of terms due to Rayleigh and Mie scattering and to molecular absorption. For molecular density \(N\) with absorption cross section \(\alpha_{abs}(x,y,\lambda) = N(x,y) \cdot \sigma(\lambda)\).

We define a projection number \(P_{ij}\) to be used as input to the tomographic reconstruction algorithm as the ratio \(\ln(P_{ij}^0/P_{ij})\), where \(P_{ij}^0\) is the normalized intensity at mirror \(M_i\). Thus

\[
P_{ij} = \int_{L_{ij}} \alpha_A(x,y,\lambda) \, ds, \tag{1}
\]

where the integral is over the path from \(M_i\) to \(D_j\).

To make a measurement, the tunable-laser beam rotates and strikes mirrors \(M_i\), which illuminate detectors \(D_j\). The projection numbers \(P_{ij}\) are measured, stored, and used to form the input to the mathematical algorithm,\textsuperscript{5} which allows the approximate reconstruction of \(\alpha_A(x,y,\lambda)\) for all \((x,y)\) in the monitored area A. To recover the pollutant density, two successive measurements are made with the laser tuned on and off the pollutant absorption, and the difference in extinction coefficients is taken to yield \(N(x,y)\sigma(\lambda)\). The reconstructed pollution field is then gray coded and displayed as a picture.

If \(\alpha_A(x,y,\lambda)\) is a continuous function \(f\), the Radon theorem guarantees that an exact reconstruction of \(\alpha_A(x,y,\lambda)\) can be found, given \textit{all} projection numbers. In practice, only a finite number of measurements can be made, and an approximation of \(\alpha_A(x,y,\lambda)\) to \(\alpha_A(x,y,\lambda)\) is formed. The closeness of the approximations \(\alpha_A(x,y,\lambda)\) to \(\alpha_A(x,y,\lambda)\) depends on the smoothness of \(f\) and on the sampling numbers \(n\) and \(m\).

Simulations\textsuperscript{3,4,6} made for the problem of designing an x-ray tomographic device for human-body sections...
involved the use of superpositions of several ellipses to simulate body parts. These simulations should also serve well for the air-pollution case because, if $Q$ is a source of pollutant, say a chimney or leak, then by the time the pollutant has attained a height $h$ in the measurement plane, the pollutant source cloud has assumed a circular shape because of diffusion; if there is a wind field, the cloud has assumed an elliptical shape.

Based on the simulations and Nyquist's theorem, it is a useful rule of thumb that, to detect clearly a circular pollutant cloud, which may be only a few per cent more absorbing than the surrounding air, it is necessary that at least two or three line integrals from each fan of measurement cross the cloud. Thus, assuming that the two rays adjacent to the central ray are just tangent to a circular pollutant cloud of radius $r = 10$ m, where the circle $A$ has radius $R = 1000$ m, we find that $m \sim 2\pi/2 \sin^{-1}(10/1000) = 314$ detectors are needed. Since for this application the mirrors $M_i$ are mounted on poles or buildings and it is natural to mount the detectors adjacent to $M_i$, we take $n = m$.

We can estimate the required transmitted laser power needed to carry out the tomographic measurement by setting the power received at the detector equal to the minimum detectable power at a given signal-to-noise ratio. Conservation of energy gives us the following expression for the power received at detector $D_j$ (Ref. 2):

$$P_j = K P_0 \frac{\sqrt{\Delta}}{\pi L_{ij}} \exp \left[ - \int_{L_{ij}} \alpha_A(x,y,\lambda)ds \right], \quad (2)$$

where $K$ is the system's optical efficiency, $P_0$ and $P_j$ are the transmitted and received powers, $A$ is the receiver aperture area, and $L_{ij}$ is the path length. Here we have assumed that mirror $M_i$ fans the incident laser beam uniformly over a $180^\circ$ field in the plane and that the circular receiver aperture is large enough to accept the diffraction-limited laser beam out of the plane. Under these assumptions, the fraction of power collected by the receiver is $\sqrt{\Delta/\pi L_{ij}}$.

For a dark-current-limited detector likely to be used in the infrared, the minimum detectable power at a signal-to-noise level $S/N$ is

$$P_{\text{min}} = \text{NEP}(S/N)\sqrt{2\Delta f}, \quad (3)$$

where $\Delta f$ is the detector or electronics bandwidth and $\text{NEP} = \sqrt{\alpha/D^*}$ is the detector noise equivalent power, which is related to the detectivity, $D^*$, by the square root of the detector area $\alpha$.

By setting $P_j = P_{\text{min}}$, we determine that the required transmitted power for the measurement at a given $(S/N)$ level is

$$P_{\text{req}} = \frac{\text{NEP}(S/N)\sqrt{2\Delta f}}{K \frac{\sqrt{\Delta}}{\pi L_{ij}} \exp \left[ - \int_{L_{ij}} \alpha_A(x,y,\lambda)ds \right]} \cdot \quad (4)$$

For a pulsed-laser source, the required transmitted energy is found by multiplying Eq. (4) by $\tau$, the system response time, and assuming that $\Delta f = 2$, to give

$$E_{\text{req}} = \frac{2\sqrt{\tau} \text{NEP}(S/N)}{K \frac{\sqrt{\Delta}}{\pi L_{ij}} \exp \left[ - \int_{L_{ij}} \alpha_A(x,y,\lambda)ds \right]} \cdot \quad (5)$$

Note that the required power and energy scale linearly with $L_{ij}$ rather than with $L_{ij}^2$, as in the differential-absorption or topographical target-measurement case. Also note that the detector bandwidth can be reduced to lower the cost of the detector and digitizing electronics without increasing the required laser power significantly. Here we have also made the simplifying assumption that the transmitted power is not significantly depleted when tuned onto the absorption resonance. An extension to the case of significant depletion is straightforward and has been treated previously by Byer and Garbuhy.

As a first example, we assume that the measurement is made over a 2-km-diameter area with a 10-m depth resolution at $S/N = 1000$, $K = 0.1$, and $\exp \int_{L_{ij}} -\alpha_A(x,y,\lambda)ds = e^{-1}$. If a time $t$ is taken to rotate the laser beam and make the measurement, then a mirror $M_i$ is illuminated for a time $t_i = W_m/(2\pi R)$, where $W_m$ is the mirror width. The detector must operate at a bandwidth $\Delta f = 2/t$, to resolve the signal. For $t = 10$ sec, $R = 1$ km, and $W_m = 10$ cm, we find that $\Delta f = 1.2 \times 10^4$ Hz. To be conservative we assume inexpensive room-temperature detectors with $\text{NEP} = 10^{-11}$ W Hz$^{-1/2}$ operating at $10^4$-Hz bandwidth. The calculated required power and energy from Eqs. (4) and (5) is

$$P_{\text{req}} = 3.0 \text{ W} \quad \text{(cw source)},$$

$$E_{\text{req}} = 0.50 \text{ mJ} \quad \text{(pulsed source)}$$

for a 10-cm receiver diameter at a 2-km range. These power and energy levels are well within the range of available laser sources and are well below the megawatt-peak-power and 0.1-J energy levels required for depth-resolved measurements by the differential absorption method. With appropriate hardware of the type now used in advanced medical computerized tomography scanners, the function $\overline{\alpha}_A(x,y,\lambda)$ can be reconstructed in approximately 10 sec, thus permitting
a 10-m spatially resolved measurement over an area bounded by a 2-km-diameter circle every 10 sec with the pulsed-laser source operating at 30 pulses/sec.

As a second example, consider a 10-km-radius circle with a spatial resolution of 100 m and a measurement time of 10 sec. The illumination time for each detector is \( t_i = 1.6 \times 10^{-5} \) sec, so that \( \Delta f = 1.2 \times 10^5 \) Hz. For a liquid-N\(_2\)-cooled detector with NEP = \( 10^{-12} \) W Hz\(^{1/2}\) and \( \alpha_A = 0.1 \) km\(^{-1}\), a value typical in the infrared, we find that

\[
P_{req} = 25 \text{ W} \quad (\text{cw source}),
\]
\[
E_{req} = 0.4 \text{ mJ} \quad (\text{pulse source})
\]

for a receiver diameter of 10 cm at a 20-km range. In this case, the improved detector sensitivity offset the order-of-magnitude increase in range. The required cw source power can be reduced by increasing the receiver aperture or the measurement time.

In the first example, the transmitted laser energy is almost two orders of magnitude below that required for a depth-resolved differential absorption measurement at 1-km range.\(^7\) In the second case, a depth-resolved absorption measurement in the infrared would require over six orders of magnitude more transmitted energy and thus be practically infeasible. These examples illustrate the advantage of the \( \sqrt{A/L_{ij}} \) scaling dependence for this measurement method.

The examples clearly show that the required transmitted laser power and energy for computerized air-pollution tomography are well within the present tunable-laser-state of the art and also well within accepted eye-safety levels. In a number of monitoring situations, the disadvantage of erecting 300 mirror-detector locations around a perimeter may be more than offset by the capability of computerized air-pollution tomography to provide a spatially resolved two-dimensional map of numerous potential pollutant sources.

In conclusion, we have shown that two-dimensional air-pollution monitoring over a wide area is possible with a cw or low-energy pulsed tunable-laser source and multiple-mirror virtual sources using conventional computerized tomography geometry and inversion methods.

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References