The Effect of Uncertain Labor Income and Social Security on Life-cycle Portfolios

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Abstract
This paper examines how labor income volatility and social security benefits influence life-cycle household portfolios. We examine how much the individual saves, and where, taking into account liquid financial wealth and annuities, and stocks versus bonds. Higher labor income uncertainty and lower old-age benefits boost demand for stable income in retirement, but also when young. In addition, a declining equity glide path with age is appropriate for the worker with low income uncertainty but for the high income risk worker, equity exposure rises until retirement. We also evaluate how changes in social security benefits influence retirement risk management.
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Risky labor income and pension payouts are key determinants of retirement wellbeing and investment behavior over the life-cycle. In the past, defined benefit (DB) pension plans and social security benefits provided a substantial and relatively stable component of retirement wealth, whereas more recently, labor market flexibility has grown, along with defined contribution (DC) plans. As a result, households will be required to take on increased responsibility for retirement accumulation and decumulation in a more uncertain world. This chapter examines how consumers can optimally allocate their saving among two major asset classes, namely equity and bonds, and two types of retirement assets, namely liquid saving and illiquid annuities. We illustrate how incorporating labor income risk as well as social security benefits influences optimal asset allocation, in a realistically-calibrated dynamic life-cycle model.

Our study extends prior literature by taking into account life annuities which pay a defined stream of benefits over the remaining lifetime (e.g. Cocco, Gomes, and Maenhout 2005). This work evaluates the impact of alternative empirical labor income trajectories, allowing for an observed inverted-U-shaped age-related pay profile as well as volatility around that profile. We use this approach to assess how differences in earnings profiles and shocks may drive life-cycle investment behavior and the demand for annuities. We also build on our own prior work (Horneff et al. 2009a, 2009b) which shows that households can benefit substantially from holding annuities as well as capital market assets. As demonstrated there, the particular appeal of annuities is that they offer consumers not only an investment return from the underlying assets but also the survival credit generated from pooling mortality risk. These papers provide insight into how a reasonable investor would optimally save and invest...
her wealth across bonds, stocks, and annuities, taking into account various levels of social security replacement rates.

In what follows, we model the effect of uncertain labor income and social security benefit replacement ratios on life-cycle portfolios. After outlining the empirical framework employed, we present results on optimal expected consumption, saving, asset allocation, and annuity purchases for representative households. We show that higher labor income risk and lower social security replacement rates will induce higher demand for stable income – not only in retirement but also earlier in life. In other words, individuals exposed to labor earnings risk can, to some extent, ‘roll their own’ personal defined benefit scheme by resorting to the private annuity market. We also show that a declining equity glide path with age is appropriate for both low and middle income risk workers, while, for the high income risk worker, equity exposure rises until retirement.

**Empirical Strategy**

To illustrate how different levels of labor income uncertainty and social security retirement benefits affect outcomes of interest, we examine the case of an individual who is assumed to work from the age of 20 until retirement at age 65, after which she expects to rely on social security benefits, withdrawals from liquid saving, and privately-purchased life annuity income.\(^2\) The maximum assumed survival age is 100. She can invest her liquid saving in riskless bonds or risky stocks, and she may also purchase immediate real fixed payout annuities, both before and after retirement. The individual anticipates surviving from period \(t\) to \(t + 1\) with probability \(p_t\), which is her subjective probability. She derives CRRA utility from consumption of a single non-durable good \(C\). The value function is maximized over the arguments \(C_{t}^{s}\), \(\pi_t\), and \(a_t\) according to \(V_t = \frac{C_{t}^{s}}{1 + \rho} + \beta p_t^{s} E_t[V_{t+1}]\) where \(\rho\) is the coefficient of relative risk aversion, \(\beta\) is a time discount factor, \(\pi_t\) is the share of liquid assets held in

\(\text{CRR}_t\), \(\pi_t\), and \(a_t\) according to \(V_t = \frac{C_{t}^{s}}{1 + \rho} + \beta p_t^{s} E_t[V_{t+1}]\) where \(\rho\) is the coefficient of relative risk aversion, \(\beta\) is a time discount factor, \(\pi_t\) is the share of liquid assets held in
equities, and \( a_t \) refers to annuity purchases each period. The household is precluded from borrowing against future labor income and from short-selling bonds, stocks, and life annuities.

A topic of considerable recent interest is how to model labor income uncertainty. We posit that each period’s labor income \( Y_t \) is given by
\[
Y_t = \exp(f(t))P_t U_t, \quad \text{with} \quad P_t = P_{t-1} N_t, \quad \text{where}
\]
\( f(t) \) represents a hump-shaped income profile over the life-cycle often used in empirical research (Cocco, Gomes, and Maenhout 2005). Here, \( P_t \) represents the permanent human capital component and \( N_t \) allows for stochastic changes; \( U_t \) is a transitory shock; \( \sigma_u \) is the volatility of the transitory shock and \( \sigma_n \) of the permanent shock. In retirement \((r > K)\), we assume (for simplicity) that the individual receives constant and real social security benefits with a constant benefit replacement ratio \((\zeta)\) with respect to final salary, expressed as
\[
Y_t = \zeta \exp(f(K))P_K, \quad \text{where} \quad K \text{ is the final year of work.}
\]

Our benchmark case for an illustrative consumer sets preference parameters to standard values found in the life-cycle literature: the relative risk aversion coefficient is \( \rho = 5 \) and the discount factor \( \beta = 0.96 \) (e.g. Gomes and Michaelides 2005). Representing subjective survival probabilities, we apply nonlinear least squares to fit the Gompertz force of mortality to the 2000 Population Basic mortality table for US females. To calculate the actuarial premium of a life payout annuity, we use annuitant mortality tables and include expense loading \( \lambda \) of 7.2 percent, consistent with Mitchell et al. (1999). The household can directly invest in two financial assets: riskless bonds and risky stocks. The riskless real bond gross return is 2 percent, while the real risky stock return is log-normally distributed with an expected return of 6 percent and a volatility of 18 percent as in much of the literature. The deterministic age-dependent labor income function for an individual with only a high-school education is taken from Cocco, Gomes, and Maenhout (2005). The assumed base case social security replacement rate \( \zeta \) of 68 percent is currently typical of low-wage retirees in the
United States (Mitchell and Phillips 2006). In an alternative scenario, we also show how outcomes would change with a lower replacement rate of 50 percent; this figure is consistent with replacement ratios for low-wage earners in Japan (OECD 2009: 39). These two alternatives are of interest given that Japan’s current demographic situation and social security shortfalls may well presage the future for the United States.

The labor income volatility parameters for the base case are $\sigma_n = 0.05$ and $\sigma_u = 0.075$, representing a labor income profile with relatively low risk; here correlations between the stock returns and the permanent (transitory) income shocks are set to $\phi_n = 0 (\phi_u = 0)$. In the alternative scenario, we evaluate results using a much higher labor income risk volatility of four times the base level as well as $\phi_n = 0.25$. In sensitivity analysis, we also show results for lower ($\rho = 3$) and higher ($\rho = 8$) levels of risk aversion. Table 1 summarizes model parameters.

Table 1 here

**Life-cycle Patterns of Investment, Saving, and Consumption**

The base case investor is one with low labor income risk, high social security replacement rate, and medium risk aversion, who faces incomplete private markets for immediate annuities with high loadings. Results appear in Figure 1, which displays the expected development of labor income, consumption, liquid saving, annuity purchases, and annuity income from age 20 to 100. To generate a smooth lifetime consumption path, the worker saves from her 30s to her mid-50s so as to pay for later consumption. By her late 50s, liquid assets – outside of annuities – rise to a maximum of almost 11 times her starting or initial labor income. After that, the investor gradually starts drawing down assets to compensate for declining labor income; after about age 60, liquid assets are deployed to buy private annuities. From retirement at age 65, liquid assets are depleted rapidly permitting the
retiree to maintain pre-retirement consumption; as well, she relies increasingly on income flowing from private annuities, which she continues to buy even well beyond retirement age. In her early 80s, her liquid assets are fully exhausted because she has no bequest motive; after that point, the annuity payout stream is considerable, helping maintain a smooth consumption path throughout her remaining lifetime.

*Figure 1 here*

Figure 2 indicates how liquid assets as a multiple of first-year labor income, and saving rates are expected to change with age. The saving rate is defined as

\[ SRate_t = 1 - C_t / Income_t , \]

and *Income* refers to the flow of labor income and annuity benefits.7 The base case is represented by the solid line, for the low labor income risk/high social security replacement rate scenario. In this safer world, the individual has little need to save early in life: the saving rate is low at young ages and only from the mid-20s does saving rise, with a peak in the mid-40s of 10 percent per year, and it becomes negative (-5 percent) in the mid-50s. After retirement, saving rates drop precipitously and reach -40 percent so as to smooth consumption; the elderly older than about age 85 have a saving rate of zero. Liquid assets grow slowly early in life, rising to around 10 times first-year labor income at their peak.

*Figure 2 here*

In the riskier scenario, the social security replacement rate is reduced to 50 percent, and labor income risk is four times the base level and correlated with the stock market. Here, the young adult will engage in substantially higher saving to build a buffer against high labor income volatility – over 40 percent per annum. Assets rise to 40 times first-year income by the late 50s. Next, the saving rate falls, crossing the zero mark around the early 60s; thereafter it rises again briefly just before retirement to offset low social security benefits. As we shall show next, the money is used mainly to purchase annuities that provide a secondary stable income stream in retirement. Nevertheless, this individual’s liquid saving continues to rise
through the mid-50s, due to returns on investment. After retirement, at 65, when the social security benefit begins, assets are consumed and the saving rate trajectory follows the one described earlier.

Turning to Figure 3, the three Panels illustrate how the individual will optimally allocate her total financial wealth by age across equities, bonds, and private annuities; here, total financial wealth is defined to include both liquid assets and the present value of future private annuity income claims. On average, as is indicated in Panel A, the base case individual (solid line) will optimally hold only equities from youth to about age 50 (apart from a few bonds early on, shown in Panel B). This is a common result in many life-cycle studies and occurs because, early in life, the individual’s main asset is human capital which has bond-like payout characteristics. This is especially true with low labor income uncertainty, where her relatively safe labor income is also protected in retirement with a relatively high social security benefit. She will optimally diversify her complete portfolio, which includes both the present value of her human capital (labor income plus social security benefits) as well as financial assets, by holding little to nothing in bonds and instead invests entirely in equities. Beginning about age 55, her asset allocation begins to include more bonds as her remaining work years dwindle.

*Figure 3 here*

The optimal pattern for privately-purchased annuities is illustrated in Panel C; these play no role in the investor’s portfolio prior to the age of 59, as they are relatively unattractive *vis a vis* bonds due to high loading and the use of annuitant survival tables in pricing. At older ages, however, the annuity survival credit rises above the bond rate; as a result, annuities crowd out bonds. In retirement, liquid assets are depleted to support consumption and buy annuities, which provide a secure private income stream (assuming no bequests). From the early 80’s, the retiree is fully annuitized.
These results should be contrasted to those generated by the alternative scenario with high labor income uncertainty and low social security replacement rates. In general we would expect that this consumer will need to save more, and hold more safe assets. This is borne out in Panels A-C (dashed line), where the equity fraction starts at zero and even in middle age remains below 30 percent; by contrast the bond fraction starts out at 100 percent and falls as the worker nears retirement. This is because labor income no longer produces a bond-like stream of payments that previously pushed the young investor into equities. This investor also demands more annuities, beginning around age 45, to help offset variable work earnings. The annuitized fraction then rises quickly around retirement, to help offset the now-low social security replacement rates. Around age 60, the rising survival credit dominates the annuity pricing offsets due to loads and the use of annuitant mortality tables. In all cases the transition to annuities is accomplished by a substantial movement out of bonds: her allocation to bonds drops from 65 to about 15 percent. As in the base case, the individual is fully annuitized from age 83 onward. It is also worth noting that, around age 65 when she becomes entitled to social security benefits, she is no longer exposed to labor income risk. Accordingly, the stable private annuity income plus the social security payments permits the investor to hold more equities. Thus equity exposure rises to over 40 percent, and then it gradually declines in favor of annuities, until liquid savings are exhausted at age 83.

Sensitivity Analysis

Next we review how results change across six combinations of labor income risk levels and social security replacement rates; comparative results appear in Table 2. Panel A focuses on the high social security replacement rate scenario and reports liquid assets, saving rates, and asset allocation fractions, for three workers: the low income volatility case (the base case above), a high volatility case (the alternative scenario defined above), and a middle risk
case (defined as $\sigma_n = 0.10$, $\sigma_u = 0.20$ and $\phi_n = 0$). Panel B indicates the same results for a lower social security benefit replacement rate of 50 percent. We report patterns for five decades of life from age 45 to 85.

*Table 2 here*

Comparing Panels A and B for all three levels of labor income risk, it is evident that the Panel B individual (with lower old-age benefits) accumulates more liquid assets (row 6 versus row 1) by saving more early in life (row 7 versus row 2) but she draws down her assets more quickly after retirement, so as to preserve her consumption stream. Now, moving horizontally across the table, irrespective of social security benefit levels, as labor income risk rises, so too do liquid assets. For example, even by age 45, the high labor income risk individual has amassed assets five times more due to a much higher saving rate (17.4 percent versus 8.5 percent).^10^ Moving horizontally across the table, it is clear that more labor income risk reduces the demand for equities (rows 3 and 8), irrespective of the social security replacement rate. Similarly, higher income risk makes annuities more attractive at younger ages, in all cases (rows 5 and 10). As seen above, a declining equity glide path with age is appropriate for both low and middle income risk workers, while, for the high income risk worker, equity exposure optimally rises until retirement. Moving down the table, we note that lower replacement rates prompt lower equity holdings for both low and middle income risk workers, while bonds and annuities become more desirable. For those facing high labor income risk, asset allocation patterns are less sensitive to old-age benefit levels, though the direction is similar.

Next we investigate the sensitivity of our results with respect to the investor’s level of risk aversion. In addition to the base case risk parameter ($\gamma = 5$) analyzed above, Table 3 tabulates results for low ($\gamma = 3$) and high ($\gamma = 8$) relative risk aversion levels, assuming the worker has a medium labor income volatility (so she is exposed to risk other than through the
capital market). As before, Panel A presents patterns of liquid saving and saving rates, as well as allocations to equities, bonds, and annuities for the high social security replacement rate; Panel B provides results for the lower value.

Table 3 here

Here, as risk aversion rises (moving horizontally across the table), liquid assets and expected saving rates again rise (rows 1, 2, 6, and 7), irrespective of the social security replacement rate; that is, higher risk aversion enhances the appeal of saving. Moving down the table, when social security benefits are reduced at a given level of labor uncertainty, higher saving rates and liquid assets are observed at younger, but not at older, ages. Also, as we move to the right in the table, as risk aversion rises, it is evident and unsurprising that the fraction in equities falls in favor of bonds and annuities. Going down a column, when the replacement rate drops, again the equity fraction falls – though it is interesting that at age 85, the least risk averse consumer still holds 15 percent of her portfolio in equities. In all three cases, bonds dominate at younger ages, while annuities crowd out bonds at older ages.

Conclusions

Retirement risk management is likely to become increasingly important with global demographic aging, a phenomenon already requiring social security benefit cuts in some developed nations such as Japan. This chapter illustrates how increasing labor income risk and reductions in social security replacement rates would be predicted to influence saving, life-cycle portfolio asset demand, and purchases of payout annuities. Our model shows that higher labor income risk and lower social security replacement rates boost saving rates early in life and liquid assets accumulated for precautionary purposes. A more uncertain and less generous environment also induces greater demand for protection in the form of stable income – early in life and in retirement. The enhanced need for safety is met not only with bonds, but
also with payout annuities whereas the demand for equities falls. Also, individuals who are more risk averse save more early in life, and hold fewer equities.

Our analysis offers several useful implications. For instance, financial advisers must be aware of possible future social security benefit cuts as they design optimal lifetime asset accumulation paths and portfolio allocations for younger clients. Also, the financial services industry and pension sponsors must take careful account of labor income risk when formulating recommendations for client portfolios. Thus we show that low and middle income risk workers will favor an equity glide path which declines with age, but for those facing high income risk, equity exposure should be low early in life and rise until retirement. Moreover, for those with uncertain labor income, it is desirable to purchase immediate payout annuities early in life so as to build up a second more stable stream of income. Our work underscores the need for workers to have a way to create their own ‘defined benefit plan equivalents’ with privately purchased payout annuities.

Acknowledgements

This research was conducted with support from the Pension Research Council at The Wharton School of the University of Pennsylvania. We are grateful for useful comments from Jason Scott and Ramu Thiagarajan. This is part of the NBER Program on the Economics of Aging.
Endnotes

1 For additional references to the rapidly-growing literature on realistically calibrated discrete dynamic portfolio choice models, see Horneff et al. (2009a, 2009b) and Wachter and Yogo (2009).

2 For a more detailed description of the modeling approach see Horneff et al. (2009a, 2009b); flexible hours and endogenous retirement ages are considered in Chai et al. (2009).

3 For this analysis, we assume that the household derives no utility from bequests. Hurd (1989) suggests that most bequests are accidental.

4 Notable prior studies on asset allocation that consider uninsurable labor income risk include Campbell and Viceira (2002); Heaton and Lucas (1997); Viceira (2001); Bodie et al. (2004); and Polkovnichenko (2007).

5 The logarithms of both $N_t$ and $U_t$ are normally distributed with means zero and with volatilities $\sigma_n$ and $\sigma_u$, respectively. The shocks are assumed to be uncorrelated.

6 We solve the optimization problem by backward induction in a three-dimensional state space, whereby for each grid point we evaluate the policy and value functions using Gaussian quadrature integration and cubic spline interpolation. For technical details we refer the interested reader to Horneff, Maurer, and Stamos (2008) and Horneff et al. (2009a, 2009b).

7 Here we report the saving rate as $SRate_t = 1 - \frac{E(C_t)}{E(Income_t)}$; in unreported results, we have also computed $SRate_t = 1 - \frac{E(C_t)}{Income_t}$ which is substantially lower in the high risk/low replacement rate scenario.

8 These are partial equilibrium computations, in that lower social security benefit are not offset by social security tax cuts.

9 This is because of the increasing survival credit at older ages, which raises expected annuity payouts vis a vis equities (Horneff et al. 2009a, 2009b).
In the medium labor income case, the saving rate is always lower than for the low income risk case, though liquid assets are higher in all cases. This is due to the fact that this medium income risk individual saves from a younger age, and at a higher rate (in results available on request).
References


Figure 1: Life-cycle Asset Allocation Patterns: Low Labor Income Risk and High Social Security Replacement Rate

Source: Authors’ calculations. Notes: The Figure embodies a retirement age of 65; replacement rate of 68%; annuity loading of 7.2%; moderate risk aversion ($\rho = 5$); volatility of transitory income shock = 7.5%; volatility of permanent income shock = 5%; and correlation of permanent labor income shock with equity returns= 0 (see text). Labor income, consumption, liquid assets, annuity purchases, and annuity payouts, are expressed as a multiple of first-year labor income.
Figure 2: Liquid Assets and Saving Rates for Low vs. High Labor Income Risk Levels and High vs. Low Social Security Replacement Rates

(A) Liquid Assets

(B) Saving Rates

Source: Authors’ calculations. Notes: Liquid asset (held in stocks and bonds) are expressed a multiple of first-year labor income; saving rates are the percent of labor plus annuity income saved.
Figure 3: Life-cycle Asset Allocation for Low vs. High Labor Income Risk Levels and High vs. Low Social Security Replacement Rates

Source: Authors’ calculations. Notes: Asset weights expressed as a percent of total wealth (liquid assets + present value of annuity claims).
Table 1: Behavioral and Market Parameters Employed in Empirical Analysis

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_n$</th>
<th>$\sigma_u$</th>
<th>$\phi_n$</th>
<th>$\zeta$</th>
<th>$\rho$</th>
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</thead>
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<tr>
<td>Low</td>
<td>0.05</td>
<td>0.075</td>
<td>0.00</td>
<td>0.50</td>
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<tr>
<td>Med</td>
<td>0.10</td>
<td>0.200</td>
<td>0.00</td>
<td>0.60</td>
<td>5</td>
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<tr>
<td>High</td>
<td>0.20</td>
<td>0.300</td>
<td>0.25</td>
<td>0.68</td>
<td>8</td>
</tr>
</tbody>
</table>

Source: Authors’ compilation.
Notes: $\sigma_n$ and $\sigma_u$ refer respectively to the volatility of the permanent and transitory income shocks; $\phi_n$ is the correlation of labor income risk with stock returns; $\zeta$ is the social security replacement rate; and $\rho$ is the coefficient of relative risk aversion coefficient.
Table 2: Life-cycle Saving Rates and Portfolio Mixes for Alternative Labor Income Risk Levels and Social Security Replacement Rates

<table>
<thead>
<tr>
<th>Panel</th>
<th>Low Labor Income Risk</th>
<th>Medium Labor Income Risk</th>
<th>High Labor Income Risk</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Age</td>
<td></td>
<td>Age</td>
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<tr>
<td></td>
<td>45 (1)</td>
<td>55 (2)</td>
<td>65 (3)</td>
</tr>
<tr>
<td></td>
<td>65 (6)</td>
<td>55 (7)</td>
<td>65 (8)</td>
</tr>
<tr>
<td></td>
<td>75 (11)</td>
<td>65 (12)</td>
<td>65 (13)</td>
</tr>
<tr>
<td></td>
<td>85 (16)</td>
<td></td>
<td></td>
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**Panel A: High Social Security Replacement Rate (68%)**

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Age</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Assets</td>
<td>11.1</td>
<td>17.9</td>
<td>12.9</td>
<td>5.7</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Saving Rate (%)</td>
<td>5.9</td>
<td>-11.0</td>
<td>-41.5</td>
<td>-20.0</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Equities (%)</td>
<td>92.8</td>
<td>75.6</td>
<td>59.3</td>
<td>34.9</td>
<td>0.0</td>
<td></td>
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<tr>
<td>Bonds (%)</td>
<td>7.2</td>
<td>24.3</td>
<td>1.3</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Annuities (%)</td>
<td>0.0</td>
<td>0.1</td>
<td>39.4</td>
<td>65.1</td>
<td>100.0</td>
<td></td>
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**Panel B: Low Social Security Replacement Rate (50%)**

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Age</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Assets</td>
<td>12.4</td>
<td>20.4</td>
<td>12.2</td>
<td>5.3</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Saving Rate (%)</td>
<td>8.4</td>
<td>-6.6</td>
<td>-42.6</td>
<td>-20.1</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Equities (%)</td>
<td>88.9</td>
<td>66.3</td>
<td>46.3</td>
<td>27.5</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Bonds (%)</td>
<td>11.1</td>
<td>33.6</td>
<td>1.2</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Annuities (%)</td>
<td>0.0</td>
<td>0.1</td>
<td>52.5</td>
<td>72.5</td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors’ calculations. Notes: Model assumes a retirement age of 65; annuity loading of 7.2%; moderate risk aversion ($\gamma = 5$). For the case of low labor income risk, the volatility of transitory income shock is 7.5%; of the permanent income shock is 5% and the correlation of the permanent shock with equity returns is 0. For the medium labor income risk case, the volatility of transitory income shock is 15%, of the permanent income shock is 10%; the correlation of the permanent shock with equity returns is 0. For the high labor income risk case, the volatility of transitory income shock is 30%, of the permanent income shock is 20%; the correlation of the permanent shock with equity returns is 0.25. Investment weights are computed as a fraction of total wealth (liquid assets + present value of annuity claims); liquid assets are computed as a multiple of initial labor income; saving rates are the percent of labor plus annuity income saved.
## Table 3: Life-cycle Saving Rates and Portfolio Mixes for Alternative Levels of Risk Aversion and Social Security Replacement Rates

<table>
<thead>
<tr>
<th>Age</th>
<th>Low Risk Aversion (γ = 3)</th>
<th>Med. Risk Aversion (γ = 5)</th>
<th>High Risk Aversion (γ = 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45</td>
<td>55</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

### Panel A: High Social Security Replacement Rate (68%)

| Liquid Assets | 6.5 | 11.4 | 14.3 | 7.2 | 0.8 | 11.1 | 17.9 | 12.9 | 5.7 | 0.0 | 15.6 | 22.0 | 8.4 | 3.8 | 0.0 |
| Saving Rate (%) | 5.7 | -7.4 | -69.8 | -48.3 | -15.6 | 5.9 | -11.0 | -41.5 | -20.0 | 1.8 | 7.8 | -10.9 | -17.8 | -6.0 | 6.2 |
| Equities (%) | 100.0 | 99.3 | 94.7 | 74.5 | 29.3 | 92.8 | 75.6 | 59.3 | 34.9 | 0.0 | 49.3 | 39.2 | 32.6 | 19.3 | 0.0 |
| Bonds (%) | 0.0 | 0.6 | 0.4 | 0.0 | 0.0 | 7.2 | 24.3 | 1.3 | 0.0 | 0.0 | 50.7 | 60.7 | 0.5 | 0.0 | 0.0 |
| Annuities (%) | 0.0 | 0.0 | 4.9 | 25.5 | 70.7 | 0.0 | 0.1 | 39.4 | 65.1 | 100.0 | 0.0 | 0.1 | 66.9 | 80.7 | 100.0 |

### Panel B: Low Social Security Replacement Rate (50%)

| Liquid Assets | 7.6 | 14.2 | 16.9 | 7.6 | 0.8 | 12.4 | 20.4 | 12.2 | 5.3 | 0.0 | 16.4 | 23.8 | 7.9 | 3.5 | 0.0 |
| Saving Rate (%) | 8.0 | -4.0 | -98.2 | -54.1 | -16.2 | 8.4 | -6.6 | -42.6 | -20.1 | 1.4 | 10.2 | -6.2 | -18.2 | -6.0 | 6.3 |
| Equities (%) | 100.0 | 98.1 | 84.5 | 56.7 | 15.0 | 88.9 | 66.3 | 46.3 | 27.5 | 0.0 | 46.4 | 35.1 | 26.9 | 16.1 | 0.0 |
| Bonds (%) | 0.0 | 1.9 | 1.0 | 0.0 | 0.0 | 11.1 | 33.6 | 1.2 | 0.0 | 0.0 | 53.6 | 64.8 | 0.3 | 0.0 | 0.0 |
| Annuities (%) | 0.0 | 0.1 | 14.6 | 43.3 | 85.0 | 0.0 | 0.1 | 52.5 | 72.5 | 100.0 | 0.0 | 0.1 | 72.8 | 83.9 | 100.0 |

Source: Authors’ calculations. Notes: Results are computed for the medium labor income risk case; here the volatility of transitory income shock is 15% and for the permanent income shock 10%; and the correlation of the permanent shock with equity returns is 0. For other definitions see Table 2.