# STRATEGIC TECHNOLOGY CHOICE AND CAPACITY INVESTMENT UNDER DEMAND UNCERTAINTY<sup>\*</sup>

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#### Abstract

This paper studies the impact of competition on a firm's choice of technology (product-flexible or product-dedicated) and capacity investment decisions. Specifically, we model two firms competing with each other in two markets characterized by price-dependent and uncertain demand. The firms make three decisions in the following sequence: choice of technology (technology game), capacity investment (capacity game) and production quantities (production game). The technology and capacity games occur while the demand curve is still uncertain, and the production game is postponed until after the demand curve is revealed.

We develop best-response functions for each firm in the technology game and compare how a monopolist and a duopolist respond to a given flexibility premium. We show that the firms may respond to competition by adopting a technology which is the same as or different from what the competitor adopts. We conclude that contrary to popular belief, flexibility is not always the best response to competition – flexible and dedicated technologies may coexist in equilibrium. We demonstrate that as the difference between the two market sizes increases, a duopolist is willing to pay less for flexible technology, whereas the decision of a monopolist is not affected. Further, we find that a firm that invests in flexibility benefits from a low correlation between demands for two products, but the extent of this benefit differs depending on the competitor's technology choice. Our results indicate that higher demand substitution may or may not promote the adoption of flexibility under competition, whereas it always facilitates the adoption of flexibility without competition. Finally we show that, contrary to intuition, as the competitor's cost of capacity increases, the premium a flexible firm is willing to pay for flexibility decreases.

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# 1 Introduction

Flexible manufacturing systems (FMS) allow a firm to cope better with demand uncertainty when manufacturing several products in the same facility by cutting production of goods for which demand is low while increasing production of goods for which demand is high.<sup>1</sup> Another advantage of FMS is the ability to respond strategically to competition (Fine 1993). Our aim in this paper is to evaluate the strategic value of manufacturing flexibility in an uncertain environment and to understand whether the value of flexibility increases or decreases under competition.

While the extant literature has showcased flexible manufacturing both as a strategic competitive edge and as a hedge against uncertainty (Roller and Tombak 1991, Fine 1993, Gerwin 1993), normative models that analyze the benefits of product flexibility and aid in decision-making under both uncertainty and competition are nonexistent. At the same time, there are many practical situations in which both competition and demand uncertainty affect decisions to invest in FMS. Upton (1995) studies 61 plants in the paper industry in which products are quite comparable across manufacturers (e.g., letter-size paper) and finds that, although the same fundamental processes are used everywhere, some firms have adopted flexible manufacturing technology while others have not. As a result, products manufactured by different companies - and hence using different technologies - compete directly in the market, so it is plausible that a firm's technology and capacity investment decisions affect its competitors' decisions. The automotive industry is another example in which decisions to invest in manufacturing flexibility are made under intense competition and demand uncertainty in the market. US automotive manufacturers traditionally built plants that were dedicated to manufacturing a single car model, with long downtimes required to switch to a different model using the same platform, and even longer downtimes to switch to a different platform. However, recently all three major US manufacturers have invested heavily in manufacturing flexibility, citing both demand uncertainty and competitive pressures as reasons (see Holweg and Pil 2004, Mackintosh 2003). For example, Ford intends to use FMS at 75% of its assembly and powertrain plants by 2010. Today, Ford already builds five different models of F-series trucks in its Dearborn truck plant and eight distinct models on two different platforms in its Norfolk plant (McMurray 2004).

We attempt to fill the void in the literature between the analysis of manufacturing flexibility as a response to demand uncertainty and as a competitive weapon. A majority of existing papers either study monopolistic models that cannot explain to what extent a competitor's decision to invest in manufacturing flexibility drives the technology investment decision of the firm or study a competitive setting with no uncertainty.

We propose a stylized model in which each of two firms can invest in either two dedicated (cheaper) production lines or one flexible (more expensive) production line. Independent of the technology choice,

<sup>&</sup>lt;sup>1</sup>For the purpose of this paper, when we refer to FMS we imply product flexibility or mix flexibility, i.e., the ability to manufacture different products on the same production line. Other types of flexibility are possible as well, e.g., volume flexibility, changeover flexibility, etc. See Gerwin (1993).

both firms manufacture both products and compete directly with each other in the two markets. Each firm makes three sequential decisions. The first is the choice of production technology (the technology game), either dedicated (D) or flexible (F). The second is capacity investment, given the choice of technology (the capacity game). These two decisions are made ex-ante, before demand curves are revealed, reflecting the long lead time involved in capacity and technology acquisition. The final decision, which is constrained by the earlier two decisions, concerns the quantities to be produced (the production game) and is expost the realization of uncertainty (responsive manufacturing). The market price is a function of the total amount of product offered in the market by the two firms (Cournot competition). We formally characterize the subgame-perfect Nash equilibria (SPNE) in the capacity and production games. Under appropriate assumptions about the demand distribution, we solve for the capacities in closed form and derive closedform expressions for expected prices and expected profits. The effect of competition on the technology choice of firms is distilled by contrasting the actions of a duopolist with those of a monopolist.

Flexibility allows a firm to take advantage of favorable states in the future; in our case, a flexible firm can take advantage of a higher demand state for one product by transferring capacity from the product with low demand to the one with high demand. A monopolist thus faces a simple choice: if the value of flexibility is high enough given an expectation of future demand realizations, a monopolist invests in flexibility, but otherwise not. We demonstrate that the choice for a duopolist is far more complicated, because he not only takes into account future demand realizations (just like a monopolist) but also needs to take into account the actions of the other firm. For instance, if the competing firm is flexible as well, both firms would then be able to appropriate higher future demand realizations, which may render flexibility less attractive to both of them. We structure our understanding of the impact of competition on the choice of technology by delineating four effects that drive the equilibrium of the technology game: the stochastic effect, the market size effect, the product substitutability effect and the cost effect. Due to the stochastic effect, each firm is willing to pay increasingly more for flexibility under higher demand uncertainty, but more important, the cost premium which a duopolist is willing to accept is higher (smaller) than the premium which a monopolist is willing to accept if the competitor invests in dedicated (flexible) technology. Flexibility is of most use when the expected demands in the two markets are approximately equal. If one market is much larger than the other, then even a flexible firm is likely to devote a bigger chunk of the capacity to manufacture the product with the (ex-ante) larger demand leaving little capacity to "move around," diminishing the ex-ante (or expected) value of flexibility. This intuition underlies the market size effect. Higher demand substitution (the product substitutability effect) amplifies both the stochastic and the market size effects. Finally, the cost effect introduces asymmetry in the willingness to pay for flexibility for the two competitors. The interplay of these four effects in our stylized model leads to the following main insights that differentiate a monopolist from a duopolist, thus distilling the impact of competition:

1. It is well known that a monopolist invests in flexible (dedicated) capacity when demand uncertainty is high (low) enough. We find that, *ceteris paribus*, this is still true under competition: each firm is willing to pay increasingly more for flexibility under higher demand uncertainty. Not surprisingly then, when demand uncertainty is low, no firm invests in flexibility, and when demand uncertainty is high, both firms invest in flexibility. However, under intermediate demand uncertainty, the stochastic and the market size effects interact to yield the following two sets of equilibria: firms either mimic their competition or choose a technology which is opposite to what the competitor adopts. The latter asymmetric equilibrium arises even if firms/markets are completely symmetric. We conclude that, contrary to popular belief, flexibility is not always the best response to competition.

2. We show that one of the important drivers of technology adoption is the difference between the mean demands for the two markets (the market size effect). This is contrary to a monopolist's technology choice that does not depend on this demand differential. As the difference between the two market sizes increases, a duopolist is willing to pay less for flexible technology, an effect that is stronger for a firm facing a dedicated competitor than it is for a firm facing a flexible competitor.

3. Extant literature has shown that a firm that invests in flexibility benefits from a low correlation between demands for two products. We confirm that this result holds under competition as well, but we also show that this effect is strongest for a flexible firm facing a dedicated competitor and weakest for a flexible firm facing a flexible competitor, whereas the impact on a monopolist lies between these two extremes. This finding is an outcome of the stochastic effect.

4. We find that the impact of product substitution differs for a monopolist and a duopolist. Namely, higher demand substitution makes flexibility more attractive for a monopolist. At the same time, higher substitutability may either increase (under a low demand differential) or reduce (under a high demand differential) the range of problem parameters under which flexibility is adopted in equilibrium.

5. Contrary to intuition, we show that as the competitor's cost of capacity (whether flexible or dedicated) increases, the premium a flexible firm is willing to pay for flexibility *decreases*. Consistent with intuition, we find that, as the firm's own cost of dedicated technology increases, the premium the firm is willing to pay for flexibility increases as well.

These findings provide immediate opportunities to test them empirically, similar to the approach in Roller and Tombak (1993), and we pursue this line of research in Goyal et al. (2006). The rest of the paper is organized as follows. Section 2 surveys related literature while emphasizing the positioning of our work. In Section 3 we formulate the stochastic three-stage game. In Section 4 we solve the technology game under appropriate assumptions, in Section 5 we discuss assumptions and limitations, and in Section 6 we summarize our findings.

# 2 Literature Survey

Two streams of literature are relevant to our study: the first explores flexibility as a hedge against demand uncertainty, and the second studies flexibility as a strategic weapon under competition. These are combined in our study, for we believe they are equally important in practice.

Papers in the first stream consider investment in flexible vs. dedicated capacity in the absence of

competition and analyze the trade-off between the higher cost of flexibility and its ability to hedge against demand uncertainty by manufacturing multiple products. All papers in this stream consider a monopolistic firm. Fine and Freund (1990) model a firm manufacturing n products within two decision epochs. In the first stage, the firm must choose the capacity levels for the n dedicated resources as well as for the one flexible resource that can manufacture all n products. In the second stage (after demand realizations) the firm decides on production quantities given the capacity constraints. Fine and Freund show that the decision to invest in flexibility is based on the cost differential between the dedicated and flexible technologies. Van Mieghem (1998) develops a similar model and finds that flexibility is beneficial even with perfect positive correlation if product margins are different. Other works that have looked at similar issues are Harrison and Van Mieghem (1998), Netessine et al. (2002) and Tomlin and Wang (2005). In all these papers, product prices are exogenous to the model. Chod and Rudi (2005) endogenize pricing decisions and analyze a firm manufacturing two products while investing in a flexible resource only. As in our paper, the capacity decision is ex-ante and the production decision is ex-post, whereas the price for a product is a function of production quantities. Chod and Rudi (2005) find that the flexible capacity and expected profits are increasing in demand variance, and that positive correlation increases capacity investment while decreasing expected profits. Further, they quantify the benefits of flexibility by comparing a firm investing in flexible technology with a firm investing in two dedicated production lines under the assumption that investment costs for the two technologies are the same. Bish and Wang (2004) consider a problem setting similar to that of Chod and Rudi (2005) but allow the firm to invest simultaneously in dedicated and flexible capacities.

The second stream of literature looks at the strategic value of flexibility in the absence of demand uncertainty. Hence, flexibility is typically shown in light of economies of scope. Fine and Pappu (1990) and Roller and Tombak (1990, 1993) model two firms competing with each other. The technology choice is modeled as a 2x2 game in strategic form. The firm investing in flexible technology can enter two markets, while the firm investing in dedicated technology enters only one market. The firms are trapped in a Prisoner's dilemma-like situation: while each can choose one market and make a monopoly profit in it, both firms invade both markets by choosing flexible technology under the threat that the rival might choose flexible technology and invade, and hence intensify competition. As a result, these papers show that flexible technology is detrimental to both firms. In addition, Roller and Tombak (1990, 1993) show that both symmetric and asymmetric equilibria can exist, that prices are lowest when both firms choose flexible technology, and that decreasing product substitutability promotes adoption of flexible technology. In Roller and Tombak, flexible technology is a prerequisite for entering the second market, whereas in our paper it is not. Hence, in our model, flexibility does not inherently fuel competition. Accordingly, prices in our model are not the lowest when both firms invest in flexible technology (they are, in fact, the highest). Moreover, decreasing product substitutability does not necessarily favor flexible technology in our work, because we model stochastic demand, which introduces new effects into the problem. In another related work, Anand and Girotra (2003) analyze the benefits of delayed differentiation under competition.

Delayed differentiation is similar to manufacturing flexibility, since it allows the firm to hedge against demand uncertainty. However, in their model firms compete in one market only while monopolizing the other market they serve.

Several other papers that do not fit into the two streams above are, nevertheless, relevant to our work. Gerwin (1993) outlines a comprehensive framework for analyzing manufacturing flexibility and describes various flexibility types. One of the first papers to explore technology adoption under competition is that of Gaimon (1989), who studies a differential game between two firms that can acquire new technology to reduce unit operating cost for a *single* product (hence, FMS is outside of the scope of this work). In their seminal paper, Jordan and Graves (1995) look at total flexibility vs. partial flexibility through the concept of chaining (in which a chain consists of product-plant links: more links correspond to higher flexibility). They find that adding limited flexibility in the right way can achieve nearly all the benefits of total flexibility in terms of hedging against demand uncertainty. Graves and Tomlin (2003) extend this work to a multi-echelon supply chain setting. Finally, Parker and Kapuscinski (2003) study the role of flexible technology in entry deterrence (in contrast, we do not model entry decisions).

We contribute to the extant literature on manufacturing/capacity flexibility by *simultaneously* studying the impact of both demand uncertainty and competitive pressures on a firm's choice of technology and attempting to bridge the gap between these two streams of literature. While our model is somewhat similar to those of Roller and Tombak (1990, 1993) and Fine and Pappu (1990), who model a two-firm two-product competitive scenario, we also incorporate demand uncertainty in the spirit of Fine and Freund (1990), Van Mieghem (1998) and particularly Chod and Rudi (2005).

# 3 The Model

Two firms are indexed by i or j; i, j = 1, 2, and  $i \neq j$ . We assume that the firms are risk-neutral and maximize expected profits (risk aversion is discussed in Section 5). Each firm manufactures two products indexed by y = 1, 2 and engages in competition with the other firm in both markets. By making the decision to enter a market exogenous to the model, we create a level playing field for the two technologies in terms of economies of scope and hence isolate flexibility as a hedge against uncertainty in a competitive environment. In each of the three stages, firms play a simultaneous-move noncooperative game with complete information. In the first stage, each firm can invest either in a flexible technology (F) that allows it to manufacture both products on the same production line or in a dedicated technology (D) for each of the products separately. The firm cannot invest in flexible and dedicated technologies simultaneously (see a discussion of this assumption in Section 5).

Depending on the technology choices in the first-stage game, three subgames can potentially emerge. The superscripts refer to the type of subgame which the firms play: (m) refers to a mixed subgame in which one firm invests in flexible and the other in dedicated technology (also referred to as the (D, F) or (F, D) subgame); (f) refers to a flexible (F, F) subgame; and (d) refers to a dedicated (D, D) subgame. The subscripts refer to the type of capacity, whether flexible (f) or dedicated (d), which can also be indexed by y for each of the products. If it is necessary to differentiate firms, the firm index i, j appears in the subscript as well.

In the second stage (the capacity game), each firm invests in a single production capacity when it adopts flexible technology and in two production capacities when it adopts dedicated technology. We denote all capacities by K, e.g.,  $K_{fi}^{f}$  is the flexible capacity of firm i in the flexible subgame. Capacity investment is costly, and we allow these costs to differ by company. We let the cost of purchasing the flexible resource be  $c_{fi}$  per unit and the cost of the dedicated resource be  $c_i$  per unit for each product for firm i, with  $c_{fi} > c_i$ , which is similar to the assumptions in Fine and Freund (1990) and several subsequent papers. The expected optimal profit of the firm in this stage is denoted by  $\Pi$  so, for example,  $\Pi_{di}^{m}$  denotes the expected profit of firm i competing in the mixed subgame and investing in two dedicated production lines with capacities  $K_{1i}^{m}$  and  $K_{2i}^{m}$ .

The last stage of the game is concerned with the quantities (denoted by q) to be put in the market given the first two decisions. This decision is ex-post because at the time of production the firm is typically better informed about market conditions. The inverse demand function for product y is  $P_y(Q_y, Q_{3-y}) =$  $A_y - Q_y - \beta Q_{3-y}$  for y = 1, 2 where  $Q_y$  is the total quantity of product y put on the market by the two firms combined (Cournot competition model with linear demand functions).  $\beta \in (-1, 1)$  is the product substitutability parameter, and  $\beta > 0$  ( $\beta < 0$ ) signifies that the products are substitutes (complements) in a Cournot game. Note that substitutability implies that the demand for a product increases with an increase in the price of the other product, and vice versa for complementarity. This demand model can be obtained from a consumer choice model (see Singh and Vives 1984) in which a representative consumer maximizes a quadratic and strictly concave utility function. For the model to be well defined, it is necessary to have  $A_y \ge \beta A_{3-y}$  and  $-1 < \beta < 1$ . The first condition implies that the domain of the probability distribution becomes a function of  $\beta$ , which further implies that the moments of the restricted demand distribution are a function of  $\beta$  as well. Similar to other papers relying on this demand model, we ignore this effect when drawing conclusions based on changes in the parameter  $\beta$ , thus assuming that these changes are so small that they do not appreciably affect moments of the probability distribution.

The quantity of product y put on the market by firm i is  $q_{yi}$  so that  $Q_y = q_{yi} + q_{yj}$ . The demand curve intercepts,  $A_y \in \Re_+$ , are random draws from a bivariate continuous distribution function F(.,.) with a density function f(.,.). We denote the mean of the marginal distribution by  $\mu_y$ , the variance by  $\sigma_y^2$  and the correlation coefficient between the two markets by  $\rho$ . Throughout the paper we use  $\sigma_T^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$ to denote the total demand uncertainty that each firm faces. We often refer to the uncertainty in demand intercepts simply as "demand uncertainty" and we also refer to  $\mu_y$  as the "expected market size" because higher  $\mu_y$  corresponds to higher expected demand for the product. We denote profits in the production game by  $\pi$ . We denote by E the expectation operator with respect to the random variables  $A_y$ , y = 1, 2. Consistent with previous literature, we normalize the marginal cost of production to zero (Fine and Pappu 1990, Roller and Tombak 1990, 1993).

## 3.1 Problem formulation

Consider the technology game in Figure 1 that is schematically represented as a  $2x^2$  matrix typical for strategic-form games (e.g., the Prisoner's dilemma). Each firm is endowed with two strategies (D and F) and each row-column intersection signifies a subgame in the technology game, while the matrix entries signify profits in the second-stage capacity game.



Figure 1. The technology game

We seek a pure strategy Nash equilibrium of this 2x2 noncooperative game. As is typical for such games, the solution is obtained by considering the best response functions of each firm given the technology choice of the other firm. Since we are unable to predict the equilibrium of the technology game up front, we proceed by analyzing capacity and production choices in all three possible subgames of the technology game. The optimization problem for a firm *i* that invests in dedicated technology for any strategic choice by the rival firm *j* is

Capacity game : 
$$\Pi_{i} = \max_{K_{1i}, K_{2i}} \{ E_{A}(\pi_{i}) - c_{i}(K_{1i} + K_{2i}) \},$$
  
Production game : 
$$\begin{cases} \pi_{i} = \max_{q_{1i}, q_{2i}} \sum_{y=1}^{2} \left[ \left( A_{y} - (q_{yi} + q_{yj}) - \beta \left( q_{(3-y)i} + q_{(3-y)j} \right) \right) q_{yi} \right], \\ s.t. \ 0 \le q_{yi} \le K_{yi}, \ y = 1, 2. \end{cases}$$

The optimization problem for a firm i that invests in a flexible technology for any strategic choice by the rival firm j is

Capacity game : 
$$\Pi_i = \max_{K_{fi}} \{ E_A(\pi_i) - c_{fi}(K_{fi}) \},$$
  
Production game : 
$$\begin{cases} \pi_i = \max_{q_{1i}, q_{2i}} \sum_{y=1}^2 \left[ \left( A_y - (q_{yi} + q_{yj}) - \beta \left( q_{(3-y)i} + q_{(3-y)j} \right) \right) q_{yi} \right], \\ s.t. \ q_{1i} + q_{2i} \le K_{fi}, \ q_{yi} \ge 0, \ y = 1, 2. \end{cases}$$

In the last stage, firms play a constrained Cournot duopoly game and, depending on the combination of the capacity and technology decisions made in the previous stages, a variety of situations have to be analyzed. In the Technical Appendix (see Goyal and Netessine 2006) we show that there always exists a unique SPNE in the production game and we solve for production quantities  $q_{1i}$ ,  $q_{2i}$  in closed form. Furthermore, we show that price nonnegativity constraints are satisfied at equilibrium and hence are omitted in the formulation. Next, we obtain closed-form expressions for  $\pi_i$ 's in each subgame, which we then use to find optimal capacity investment decisions. The optimal capacities are defined implicitly through complex optimality conditions, and therefore closed-form expressions for the  $\Pi_i$ 's cannot be obtained without additional assumptions.

# 4 The technology game

The analysis of the technology game, which is the main thrust of this paper, is obscured by the intractability of the problem in its current form. Although optimality conditions for capacities are available, they define capacities implicitly rather than explicitly, and profit expressions are available only as functions of these implicitly defined capacities. Even assuming a specific distribution for the demand intercepts (e.g., a uniform one) does not lead to tractable expressions for either capacities or profits in the capacity game (see Chod and Rudi 2005 for a similar analysis in a monopolistic setting). In order to simplify the problem we impose two assumptions (which are in the spirit of Chod and Rudi 2005) below, and discuss their justifications and the impact of relaxing them in Section 5.

Assumption A-1: The flexible firm always manufactures both products or  $q_{yi} > 0$ , y = 1, 2.

Assumption A-1 essentially requires that the demand intercept realizations are never too high for one product so as to render the other product uneconomical to manufacture for a flexible firm.

**Assumption A-2**: Each firm produces to capacity: for a dedicated firm  $q_{yi} = K_{yi}$ , y = 1, 2 and for a flexible firm  $q_{1i} + q_{2i} = K_{fi}$ .

Assumption A-2 is known in the literature as *clearance* (i.e., a firm sets prices so as to clear the market) as opposed to *holdback* (i.e., a firm may hold back some capacity after realizing the demand curve) and it essentially renders trivial decisions in the last stage (the production game) for the firm investing in dedicated technology. However, the firm investing in flexible technology still has to allocate its capacity to each of the two products. Hence, production decisions for the flexible firm are not trivial, and the capacity and production games still need to be considered separately in the mixed and flexible subgames.

We now obtain closed-form solutions for capacity and profit for each of the three subgames using Assumptions A-1 and A-2. Since these solutions are unique, it follows that there is a unique equilibrium in the capacity game for all three subgames. Proofs for all propositions are at the end of the paper.

**Proposition 1** Equilibrium capacities and profits in the three technology subgames are characterized in Tables 1 and 2 as follows with firm i as the row player and firm j as the column player. In each cell, the

first entry is for firm i and the second for firm  $j^2$ :

$\begin{array}{ c c c c c c }\hline & D & F \\ \hline & D & \frac{\mu_1 + \mu_2 + 2c_j - 4c_i}{3(1+\beta)} & \frac{\mu_1 + \mu_2 + 2c_{fj} - 4c_i}{3(1+\beta)} \\ \hline & \mu_1 + \mu_2 + 2c_i - 4c_j & \frac{\mu_1 + \mu_2 + 2c_i - 4c_{fj}}{3(1+\beta)} \\ \hline & F & \frac{\mu_1 + \mu_2 + 2c_j - 4c_{fi}}{3(1+\beta)} & \frac{\mu_1 + \mu_2 - 4c_{fi} + 2c_{fj}}{3(1+\beta)} \\ \hline & \mu_1 + \mu_2 + 2c_{fi} - 4c_j & \frac{\mu_1 + \mu_2 - 4c_{fj} + 2c_{fi}}{3(1+\beta)} \\ \hline & \frac{\mu_1 + \mu_2 + 2c_{fi} - 4c_j}{3(1+\beta)} & \frac{\mu_1 + \mu_2 - 4c_{fj} + 2c_{fi}}{3(1+\beta)} \\ \hline \end{array}$		
$ \begin{array}{c ccccc}  & \underline{\mu_{1} + \mu_{2} + 2c_{j} - 4c_{i}} \\  & \underline{\mu_{1} + \mu_{2} + 2c_{j} - 4c_{j}} \\  & \underline{\mu_{1} + \mu_{2} + 2c_{i} - 4c_{j}} \\  & \underline{\mu_{1} + \mu_{2} + 2c_{i} - 4c_{j}} \\  & \underline{\mu_{1} + \mu_{2} + 2c_{j} - 4c_{fi}} \\  & \underline{\mu_{1} + \mu_{2} + 2c_{j} - 4c_{fi}} \\  & \underline{\mu_{1} + \mu_{2} + 2c_{fi} - 4c_{j}} \\  & \underline{\mu_{1} + \mu_{2} - 4c_{fi} + 2c_{fi}} \\  & \underline{\mu_{1} + 2c_{fi} + 2c_{fi}} \\  & \underline{\mu_{1} + 2c_{fi} + 2c_{fi}} \\  & \underline{\mu_{1} + 2c_{fi} + 2c_{fi}} \\  & \mu_{1$	D F	
$\mathbf{F} \begin{bmatrix} \frac{\mu_1 + \mu_2 + 2c_j - 4c_{fi}}{3(1+\beta)} & \frac{\mu_1 + \mu_2 - 4c_{fi} + 2c_{fj}}{3(1+\beta)} \\ \frac{\mu_1 + \mu_2 + 2c_{fi} - 4c_j}{3(1+\beta)} & \frac{\mu_1 + \mu_2 - 4c_{fj} + 2c_{fi}}{3(1+\beta)} \end{bmatrix}$	$\begin{array}{ c c c c c c } D & \frac{\mu_1 + \mu_2 + 2c_j - 4c_i}{3(1+\beta)} & \frac{\mu_1 + \mu_2 + 2c_{fj} - 4c_i}{3(1+\beta)} \\ \frac{\mu_1 + \mu_2 + 2c_i - 4c_j}{3(1+\beta)} & \frac{\mu_1 + \mu_2 + 2c_{fj} - 4c_{fj}}{3(1+\beta)} \end{array}$	
$O(1+\beta)$ $O(1+\beta)$	$\mathbf{F} \begin{bmatrix} \frac{\mu_1 + \mu_2 + 2c_j - 4c_{fi}}{3(1+\beta)} \\ \frac{\mu_1 + \mu_2 + 2c_{fi} - 4c_j}{3(1+\beta)} \end{bmatrix} \begin{bmatrix} \frac{\mu_1 + \mu_2 - 4c_{fi} + 2c_{fi}}{3(1+\beta)} \\ \frac{\mu_1 + \mu_2 - 4c_{fj} + 2c_{fi}}{3(1+\beta)} \end{bmatrix}$	i_

Table 1. Capacities in technology subgames

	D	$\mathbf{F}$
D	$\frac{\frac{(\mu_1 + \mu_2 - 4c_i + 2c_j)^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{18(1-\beta)}}{\frac{(\mu_1 + \mu_2 - 4c_j + 2c_i)^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{18(1-\beta)}}$	$\frac{\frac{(\mu_1+\mu_2+2c_{fj}-4c_i)^2}{18(1+\beta)}+\frac{(\mu_1-\mu_2)^2}{16(1-\beta)}}{\frac{(\mu_1+\mu_2+2c_i-4c_{fj})^2}{18(1+\beta)}+\frac{(\mu_1-\mu_2)^2}{32(1-\beta)}+\frac{\sigma_T^2}{8(1-\beta)}}$
F	$\frac{\frac{(\mu_1 + \mu_2 + 2c_j - 4c_{fi})^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{32(1-\beta)} + \frac{\sigma_T^2}{8(1-\beta)}}{\frac{(\mu_1 + \mu_2 + 2c_{fi} - 4c_j)^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{16(1-\beta)}}$	$\frac{\frac{(\mu_1 + \mu_2 - 4c_{fi} + 2c_{fj})^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{18(1-\beta)} + \frac{\sigma_T^2}{18(1-\beta)}}{\frac{(\mu_1 + \mu_2 - 4c_{fj} + 2c_{fi})^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{18(1-\beta)} + \frac{\sigma_T^2}{18(1-\beta)}}$

Table 2. Profits in technology subgames

Note that the optimal capacity is increasing in the means of the demand intercepts and in the cost of the competitor's capacity. It is also decreasing in the firm's own capacity cost and the substitutability parameter: the more substitutable the products (the higher the  $\beta$ ), the lower the investment in capacity, since demand for one product can be substituted for another. This is consistent with the findings of Roller and Tombak (1993) but different from those of Chod and Rudi (2005), because they utilize a different demand function. Because of risk-neutrality and Assumptions A1 and A2, capacity decisions do not depend on variance or correlation of demand intercepts. Under the clearance assumption, the total capacities translate directly into expected prices.

Each profit expression has up to three terms. We call the first two terms of each profit function the deterministic component and the last term the stochastic component. Note that the firm investing in dedicated technology has only the deterministic component, whereas a flexible firm has both components in any subgame. The deterministic component increases in the cost of the competitor's capacity, and decreases in the firm's own capacity cost. However, the impact of  $\beta$  on the deterministic component is less obvious. The first term of the profit expression which depends on the total expected market size ( $\mu_1 + \mu_2$ ) decreases in  $\beta$ : the more substitutable the products, the more the two markets overlap, and therefore the smaller the effective size of the total market. These findings are consistent with previous literature (Roller and Tombak 1993). However, the profit also depends on the interaction between the difference in the market size ( $\mu_1 - \mu_2$ ) and the nature of the products (substitutes/complements). For a given total market size (i.e., keeping  $\mu_1 + \mu_2$  constant) the firm prefers to maximize the difference in the two market sizes, the effect of which is amplified when products are substitutable (positive  $\beta$ ). This effect results from

<sup>&</sup>lt;sup>2</sup>Note that this solution is valid as long as problem parameters are such that  $K_i^d$ ,  $K_i^f$ ,  $K_f^m$ ,  $K_d^m \ge 0$ . From here on we assume that these restrictions hold.

the convexity of the profit in mean demand: a high demand in one of the markets and a low demand in the other market benefits the firm more than two approximately equal demands, because the firm can use responsive pricing to charge a high price in the larger market to more than compensate for the low price in the smaller market. With substitutable products, if the demand for one product increases, the demand for the other decreases, thereby reinforcing the effect of the difference in market sizes. Moreover, due to this effect a firm's profit may decrease in the mean of demand intercept  $\mu_i$  if  $\mu_i < \mu_j$ : the firm may not benefit from a larger mean demand in one market if this is the smaller market.

The stochastic component of the profit function is present only for a flexible firm. It increases in the variances of the distribution of the random demand intercepts and in the product substitutability parameter. The effect of an increase in variance on profits was first shown by Oi (1961), who demonstrated that a firm that uses responsive pricing makes more money in the high-demand state than it loses in the low-demand state. A flexible firm can allocate capacity from one market to another, thereby taking advantage of higher demand realizations, something that the dedicated firm is ill-equipped to do (hence the dedicated firm's profit does not have the stochastic component). Increasing the variance permits realizations of higher demand states, and therefore higher profits. Higher substitutability of two products amplifies the contribution of the stochastic term, because the firm is able to cope better with demand uncertainty if products can easily substitute for one another.

## 4.1 Best response functions

We are ready to characterize the best responses of the firms in the technology game. We characterize the best response of firm i to a given technology choice for firm j, and an analogous result holds for the best response of firm j. However, it is helpful to consider first how a monopolist behaves under identical circumstances; that is, what leads a monopolist to choose dedicated or flexible technology? Determining the drivers of technology choice helps us to distill the effect of competition on the choice of technology.

**Proposition 2** (i) If the monopolist invests in two dedicated production lines, his optimal expected profit is

$$\Pi_d^M = \sum_{y=1}^2 \frac{(\mu_y - c)^2}{4(1+\beta)}.$$

If the monopolist invests in one flexible production line, his optimal expected profit is

$$\Pi_f^M = \sum_{y=1}^2 \frac{(\mu_y - c_f)^2}{4(1+\beta)} + \frac{\sigma_T^2}{8(1-\beta)}.$$

(ii) The monopolist invests in a flexible technology whenever<sup>3</sup>

$$c_f \le c_{fM} = \frac{\mu_1 + \mu_2}{2} - \frac{1}{2}\sqrt{(\mu_1 + \mu_2 - 2c)^2 - \left(\frac{1+\beta}{1-\beta}\right)\sigma_T^2}.$$
(1)

As can be seen from Proposition 2, a monopolist invests in flexible technology when its cost is below a threshold  $c_{fM}$ . This threshold is increasing in total demand variance, the substitutability parameter, and the cost of the dedicated technology and is decreasing in means of demand intercepts. Unlike the monopolist, a firm under competition has two threshold curves, one when the competitor invests in flexible technology and the other when the competitor invests in dedicated technology.

**Proposition 3** (i) When firm j invests in dedicated technology, the best response of firm i is to invest in flexible technology whenever

$$c_{fi} < \bar{c}_{fi} = \frac{\mu_1 + \mu_2 + 2c_j}{4} - \frac{1}{4}\sqrt{(\mu_1 + \mu_2 - 4c_i + 2c_j)^2 + \frac{7}{16}\left(\frac{1+\beta}{1-\beta}\right)(\mu_1 - \mu_2)^2 - \frac{9}{4}\left(\frac{1+\beta}{1-\beta}\right)\sigma_T^2}.$$
 (2)

Otherwise, firm i should invest in dedicated technology.

(ii) When firm j invests in flexible technology, the best response of firm i is to invest in flexible technology whenever

$$c_{fi} < \underline{c}_{fi} = \frac{\mu_1 + \mu_2 + 2c_{fj}}{4} - \frac{1}{4}\sqrt{\left(\mu_1 + \mu_2 - 4c_i + 2c_{fj}\right)^2 + \frac{1}{8}\left(\frac{1+\beta}{1-\beta}\right)\left(\mu_1 - \mu_2\right)^2 - \left(\frac{1+\beta}{1-\beta}\right)\sigma_T^2}.$$
 (3)

Otherwise, firm i should invest in dedicated technology.

(*iii*) 
$$\bar{c}_{fi}\Big|_{\sigma_T^2=0} \leq \underline{c}_{fi}\Big|_{\sigma_T^2=0} \leq c_i.$$
  
(*iv*)  $\partial \bar{c}_{fi}/\partial \sigma_T^2 \geq \partial \underline{c}_{fi}/\partial \sigma_T^2 \geq 0, \partial^2 \bar{c}_{fi}/\partial (\sigma_T^2)^2 \geq 0, and \ \partial^2 \underline{c}_{fi}/\partial (\sigma_T^2)^2 \geq 0.$ 

The two cost thresholds in Proposition 3 are functions of the first two moments of the distributions of the demand intercepts, the product substitutability parameter and the cost profile of the firm and its competitor. To gain intuition, it is convenient to plot both thresholds for firm *i* against total demand uncertainty  $\sigma_T^2$  on the horizontal axis (Figure 2). According to part (*iii*) of the proposition, in the absence of demand uncertainty  $\underline{c}_{fi}$  starts above  $\overline{c}_{fi}$ , though by part (*iv*)  $\overline{c}_{fi}$  increases faster than  $\underline{c}_{fi}$  and both thresholds are increasing convexly in demand uncertainty. Hence, the two cost thresholds intersect at most once. While other graphical representations are possible, we believe that demand uncertainty is the most important driver of the choice of technology and therefore we emphasize its role in the picture.

Depending on the relative sizes of demand uncertainty and the cost of flexible capacity, four distinct best responses arise (Areas I through IV in Figure 2). In Area I the cost of flexibility is large enough

 $<sup>^{3}</sup>$ Note that if the expression under the square root becomes negative (e.g., for large enough demand variance), the cost threshold becomes infinite and the firm always invests in flexibility. The same stipulation applies in a competitive scenario.

 $(c_{fi} \ge \max(\bar{c}_{fi}, \underline{c}_{fi}))$  so that firm *i* invests in dedicated technology independent of the competitor's choice of technology. Hence, the only equilibria that can emerge in this area are (D, D) and (D, F), where (D, F)denotes that firm *i* selects *D* and firm *j* selects *F*. In Area II, where  $\underline{c}_{fi} \ge c_{fi} \ge \bar{c}_{fi}$ , the firm responds symmetrically to the competitor so that if the competitor's technology is flexible, firm *i* invests in flexible technology, and if the competitor's technology is dedicated, firm *i* invests in dedicated technology. Thus, either the flexible (F, F) or the dedicated (D, D) equilibrium emerges.<sup>4</sup> In Area III,  $\underline{c}_{fi} \le c_{fi} \le \bar{c}_{fi}$ , in which case the firm responds asymmetrically to the competitor's technology choice (i.e., if the competitor is flexible, firm *i* invests in dedicated technology, and if the competitor is dedicated, firm *i* invests in flexible technology). The only equilibria that can emerge in this area are asymmetric, either (D, F) or (F, D). Lastly, in Area IV,  $c_{fi} \le \min(\underline{c}_{fi}, \overline{c}_{fi})$ , and firm *i* invests in flexible technology independent of the competitor's technology choice. Hence, the only equilibria that can emerge in this area are (F, F) or (F, D).

The outcome of the technology game is formed by four thresholds, two for each competitor. To understand a variety of situations that can arise, we proceed by analyzing four effects that influence the cost thresholds: (i) the stochastic effect (i.e., the effect of the total demand uncertainty  $\sigma_T^2$ ), (ii) the market size effect (i.e., the effect of the difference in the market sizes  $\mu_1 - \mu_2$ ), (iii) the product substitutability effect (i.e., the effect of  $\beta$ ), and finally (iv) the cost effect (i.e., the effect of the cost differential between firms). In the following subsections, we detail all four effects whose interaction drives the Nash equilibrium in the technology game.



Figure 2. The best response of player i in the technology game

## 4.1.1 The stochastic effect

Because of endogenous pricing, the profit function of the flexible firm is convex in the random intercepts  $(A_1, A_2)$ , and this convexity induces a risk-seeking behavior. A flexible firm prefers higher demand realiza-

<sup>&</sup>lt;sup>4</sup>Note that, depending on where the two thresholds cross, Area II may not exist.

tions since it can allocate capacity between markets. Due to the convexity of the profit function, the firm makes more money by transferring capacity to the market with higher demand than it loses by transferring capacity from the market with lower demand. Since higher demand variance permits higher realizations of  $(A_1, A_2)$ , the profits of the flexible firm increase in variance. This effect is captured by the stochastic term of the profit function.

According to part (iv) of Proposition 3, the threshold  $\bar{c}_{fi}$  (when the competitor is dedicated) increases faster in total variance than the threshold  $\underline{c}_{fi}$  (when the competitor is flexible). Thus, as demand uncertainty increases, the firm values flexibility (marginally) more when the competitor has not invested in flexibility. We develop the following intuition behind this result. When a firm competes with a dedicated competitor, the advantage of a high demand realization in the market is appropriated entirely by the firm with flexible technology, since the dedicated firm cannot reallocate capacity expost demand realization. However, when the firm competes with a flexible competitor, both firms take advantage of the higher demand state, thereby reducing the effective spike in demand as seen by the firm in question. This, in effect, dampens the variability that the firm perceives: a spike in the size of one market due to a favorable demand realization is moderated by the competitor's response. The effective variability falls, and flexible technology becomes less valuable. Thus, faced with a dedicated competitor, the flexible firm enjoys a monopoly over the stochastic component (the stochastic component of the flexible firm in the mixed subgame is equal to the stochastic component of a monopolist), whereas when faced with a flexible competitor, the firm shares the benefits of the stochastic component (the stochastic component of the flexible firm in the flexible subgame is less than the stochastic component of a monopolist). Thus, flexibility is more useful if the competitor is *not* flexible. We call this differential impact of competition on the value of flexibility the stochastic effect. Note also that the sum of the stochastic components of the two firms in the flexible subgame is smaller than the stochastic component of the monopolist: when both firms invest in flexibility, by competing they not only decrease their respective shares of the stochastic component (their pieces of the pie) but also decrease the total value of the stochastic component (the size of the total pie).

To isolate the impact of the stochastic effect on the equilibrium of the technology game and to contrast the best response functions of the duopolist with a similar cost threshold for a monopolist, it is essential to eliminate other effects from the model. Namely, to eliminate the market size and product substitutability effects, we let  $\mu_1 = \mu_2 = \mu$  and  $\beta = 0$ . Furthermore, to remove the cost effect and enable a meaningful comparison between a duopolist and a monopolist, we let  $c_i = c_j = c$  and  $c_{fi} = c_{fj} = c_f$ . The resulting cost thresholds are

$$c_{fM}^{se} = \mu - \sqrt{(\mu - c)^2 - \frac{1}{4}\sigma_T^2},$$
  

$$\bar{c}_f^{se} = \frac{\mu + c}{2} - \frac{1}{2}\sqrt{(\mu - c)^2 - \frac{9}{16}\sigma_T^2}, \text{ and}$$
  

$$\underline{c}_f^{se} = c + \frac{\sigma_T^2}{16(\mu - c)},$$

where the superscript se stands for the stochastic effect. The following result is easy to verify:

 $\textbf{Corollary 1} \ \left| \partial \bar{c}_{f}^{se} / \partial \sigma_{T}^{2} \right| \geq \left| \partial c_{fM}^{se} / \partial \sigma_{T}^{2} \right| \geq \left| \partial \underline{c}_{f}^{se} / \partial \sigma_{T}^{2} \right|.$ 

We illustrate these cost thresholds in Figure 3<sup>5</sup> along with the resulting equilibria of the game. We observe that all three cost thresholds intersect at  $\sigma_T^2 = 0$ . This observation, coupled with the result of Corollary 1, implies that  $\overline{c}_f^{se} \ge c_{fM}^{se} \ge c_f^{se}$ , so that the threshold of a monopolist lies between the cost thresholds of a duopolist. Hence, when a firm faces a dedicated competitor, the firm invests in flexible technology for a wider range of cost parameters than it would without any competition. Another way to emphasize this effect is to say that for  $c_{fM}^{se} < c_f < \overline{c}_f^{se}$ , a firm facing no competition would not invest in flexible technology, but a duopolist would when the competitor invests in dedicated technology. On the other hand, when a firm faces a flexible competition. For example, for costs such that  $\underline{c}_f^{se} < c_f < c_{fM}^{se}$ , a firm behaves differently under competitor investing in flexible technology. Hence, due to the stochastic effect, a firm behaves differently under competition when making its choice of technology than it would without competition.



Figure 3. Nash equilibrium in the presence of the stochastic effect

To conclude the study of the stochastic effect, we note the impact of demand correlation  $\rho$  and the uncertainty in individual demands  $\sigma_y$  upon cost thresholds. The above corollary implies that both cost thresholds decrease in the correlation coefficient but also shows that the impact of an increase in correlation is strongest for the flexible firm facing a dedicated competitor and the weakest for the flexible firm facing a flexible competitor, whereas the impact for the monopolist lies between these two extremes. Hence, a flexible firm in a mixed equilibrium is most sensitive to changes in the correlation coefficient. Though the effect of correlation on flexibility has been well studied in the literature (Fine and Freund 1990, van

<sup>&</sup>lt;sup>5</sup>Because of symmetry, the cost thresholds for firm i and firm j in the figure coincide.

Mieghem 1998, etc.), the above discussion highlights the differential impact of the *type* of competition on the degree of correlation. Finally, we note (similar to Chod and Rudi 2005) that all thresholds increase in  $\sigma_y$  as long as  $\sigma_y > \rho \sigma_{3-y}$  and decrease otherwise with the speed of change similar to the result in Corollary 1.

#### 4.1.2 The market size effect

Expressions for cost thresholds (2) and (3) indicate that the best responses depend on the total expected market size  $(\mu_1 + \mu_2)$  as well as on the difference in the expected market sizes. Specifically, the dependence is through the deterministic component of the profit which is present for all technologies in all subgames, and thus differs from the stochastic effect, which is due to the stochastic component of the profit present only for the flexible firm. At the same time we note that the monopolist's cost threshold (1) depends solely on the total expected market size. We call this differential impact of market sizes on a duopolist and a monopolist the *market size effect*. To better understand it, we study the behavior of the cost thresholds in response to a change in the difference in expected market sizes, while keeping the total expected market size constant.

## **Proposition 4** For a fixed $\mu_1 + \mu_2$ , $\partial \bar{c}_{fi} / \partial (\mu_1 - \mu_2) \leq \partial \underline{c}_{fi} / \partial (\mu_1 - \mu_2) \leq 0$ .

Proposition 4 asserts that, as the difference in market sizes increases, the duopolist is willing to pay less for flexible technology (both thresholds decrease in  $\mu_1 - \mu_2$ ). Moreover, this effect is stronger for a firm facing a dedicated competitor than for a firm facing a flexible competitor and, as a result, threshold  $\bar{c}_{fi}$  decreases faster than  $\underline{c}_{fi}$ . To understand why, first note that the term proportional to  $\mu_1 - \mu_2$  has the same multiple in the dedicated and flexible subgames. That is, if firms choose the same technology (either dedicated or flexible), they simply split equally the benefits from market asymmetry. However, the situation is different in the mixed subgame. The firm investing in dedicated technology accrues two thirds of the benefits of the market size effect, because dedicated technology is better suited for appropriating the deterministic component of the market potential, whereas flexible technology is better suited for appropriating the stochastic component. When facing a flexible competitor, the firm is less affected by the market size effect because, independent of whether it invests in dedicated or flexible technology, it derives about the same profit, although the dedicated technology performs slightly better (the ratio of profits derived from the terms proportional to  $\mu_1 - \mu_2$  is 18/16). However, when a competitor is dedicated, the firm is much better off investing in dedicated technology if it wants to benefit from asymmetry between markets (the ratio of profits derived from the terms proportional to  $\mu_1 - \mu_2$  is 32/18). It is clear that the market size effect is purely an outcome of competition, since it is not present in a monopolistic scenario.

To illustrate the incremental value of the market size effect, we let  $c_i = c_j = c$ ,  $c_{fi} = c_{fj} = c_f$  and  $\beta = 0$  to isolate the stochastic and the market size effects. The set of resulting Nash equilibria is shown in Figure 4. We see that, due to the market size effect, an additional set of equilibria  $\{(D, D), (F, F)\}$  is possible,

a scenario that does not occur when  $\mu_1 = \mu_2$ . Recall that the stochastic effect pulls both cost threshold curves up, whereas the market size effect pulls them down (as illustrated in Figure 4). The outcome results from the tension between the two effects: for a low enough variance, the market size effect dominates the stochastic effect, resulting in the additional set of equilibria, whereas for a high enough variance the situation in Figure 3 emerges.



Figure 4. Nash equilibria in the presence of stochastic and market size effects

## 4.1.3 The product substitutability effect

The product substitutability effect alters the relative sizes of areas in which various equilibria occur, but it does not introduce any new equilibria in the game. Namely, the product substitutability parameter affects cost thresholds (equations 2 and 3) by amplifying both the stochastic and the market size effects. The stochastic effect is amplified for substitutable products because high demand uncertainty is easier to cope with if products are substitutable. Similarly, as we discussed after Proposition 1, the impact of the difference in the market sizes is amplified for substitutable products. Since the stochastic effect and the market size effect move cost thresholds in opposite directions, and  $\beta$  amplifies both effects, the net effect of  $\beta$  on the areas in which each equilibrium occurs is somewhat ambiguous and depends on the comparative sizes of the stochastic and market size effects. For example, if the stochastic effect dominates (e.g., for a large enough demand uncertainty), then the impact of increasing  $\beta$  is to expand areas of (F, F) and  $\{(D, F), (F, D)\}$  equilibria while shrinking the areas of (D, D) and  $\{(D, D), (F, F)\}$  equilibria. If the market size effect dominates (e.g., for highly asymmetric markets), the exact opposite happens. Independent of which effect dominates, the optimal capacities decrease in demand substitutability. Note also that, since the monopolist is not subject to the market size effect, his cost threshold unambiguously increases in  $\beta$ , thus making flexibility more attractive. Hence, competition alters the product substitutability effect.

## 4.1.4 The cost effect

The last effect we study arises due to the cost differential between the two firms. We note that, if costs are symmetric across firms (i.e.,  $c_i = c_j = c$  and  $c_{fi} = c_{fj} = c_f$  so that the cost effect is absent), then the cost thresholds (equations 2 and 3) are symmetric as well. Hence, Nash equilibria are obtained via the areas formed by the intersection of just two cost thresholds in Figure 4. However, if costs are not symmetric, then each firm has its own threshold (equations 2 and 3), and to obtain the equilibrium of the game we must consider all four cost thresholds. It is worth pointing out that the cost effect is entirely due to competition, since cost asymmetry is irrelevant in the case of a monopoly.

We now analyze the sensitivity of the cost thresholds to changes in the firm's own costs as well as to changes in the competitor's costs. The following proposition establishes this result.

**Proposition 5** (i)  $\partial \bar{c}_{fi} / \partial c_i \geq 0$ ,  $\partial \underline{c}_{fi} / \partial c_i \geq 0$ ,

 $(ii) \partial \bar{c}_{fj} / \partial c_i \leq 0, \ \partial \underline{c}_{fj} / \partial c_i = 0, \ and$ 

(*iii*)  $\partial \underline{c}_{fj} / \partial c_{fi} \leq 0, \ \partial \overline{c}_{fj} / \partial c_{fi} = 0$ .

Not surprisingly, as the firm's cost of dedicated technology increases (part (i)), the premium the firm is willing to pay for flexibility increases as well. However, the impact of the competitor's costs on the firm's thresholds is somewhat counterintuitive. As the competitor's cost *increases*, the premium a flexible firm is willing to pay for flexibility *decreases*. This is an unexpected result because, intuitively, we would expect that as flexibility becomes less attractive to the competitor, it should become more attractive to the firm. We reason as follows. In any subgame, only the first term of firms' profit functions depends on capacity costs and hence the cost effect does not alter either the stochastic or the market size effect. This first term is directly proportional to the installed capacity and reflects the firm's ability to appropriate the deterministic component of the profit. As the competitor's (firm j's) cost of technology ( $c_j$  or  $c_{fj}$ ) increases, his capacity investment decreases, thus decreasing his ability to appropriate the deterministic component of the profit function. Hence, purely from the standpoint of appropriating the increasingly lucrative deterministic component, since firm *i* faces weaker competition for this component from firm j, it prefers the less expensive dedicated technology, and the cost thresholds decrease.

We illustrate the results of Proposition 5 in Figures 5 and 6, while assuming for clarity of exposition that  $\mu_1 = \mu_2$  so that the market size effect is absent. First, we analyze the impact of the rise in the cost of dedicated technology  $c_i$ . By part (i) of Proposition 5, the cost thresholds  $\bar{c}_{fi}$  and  $\underline{c}_{fi}$  increase, and the cost threshold  $\bar{c}_{fj}$  decreases. The resulting best response curves are shown in Figure 5 where we assume that  $c_{fi} = c_{fj}$ . The cost thresholds of firm i lie above the corresponding cost thresholds of firm j. Hence, we now obtain additional areas V, VI and VII in which the equilibrium of the technology game is (F, D), whereby firm i is flexible and firm j is dedicated. In these areas, the firm whose cost of dedicated capacity is higher (firm i in this case) invests in flexibility, whereas the firm with the lower cost (firm j) invests in dedicated technology, because the firm with higher cost of dedicated technology is willing to bear a higher cost of flexibility for the same flexibility premium. Figure 6 illustrates graphically part (*iii*) of Proposition 5. For simplicity, we assume that there is no cost asymmetry in the costs of dedicated technologies, i.e.,  $c_i = c_j = c$ , and therefore  $\bar{c}_{fi} = \bar{c}_{fj}$ . We assume that  $c_{fi} > c_{fj}$ , and hence by part (*iii*) of Proposition 5,  $\underline{c}_{fj} < \underline{c}_{fi}$ , as shown in Figure 6. Moreover, we notice that because of asymmetry in the cost of flexible technology, the equilibrium of the game is found by plotting two points representing flexible capacity costs for both firms, as shown in Figure 6. Hence, in this particular case, the equilibrium of the technology game is  $\{(D, F), (F, D)\}$  so that only one of the firms invests in flexibility. We notice that, had both costs been the same (i.e.,  $c_{fi} \rightarrow c_{fj}$ , which leads to  $\underline{c}_{fj} \rightarrow \underline{c}_{fi}$ ), the equilibrium would have been (F, F). This example highlights the impact of asymmetry in the costs of flexible technology in an extreme case where the set of equilibria changes entirely from (F, F) to  $\{(D, F), (F, D)\}$ . However, in many cases (and it is easy to construct these), the equilibria may not shift at all.



Figure 5.  $c_i > c_j, c_{fi} = c_{fj}$ 

Figure 6.  $c_i = c_j = c, c_{fi} > c_{fj}$ 

## 4.2 Nash equilibrium of the technology game

As we demonstrate, compared to a monopolist, a duopolist faces a more complex decision process, and the outcome of a duopoly is governed by the interaction of four different effects.

Due to the stochastic effect, a duopolist behaves differently when facing a flexible competitor than he does when facing a dedicated competitor. In the presence of the stochastic effect alone, the possible equilibria of the technology game are  $\{(D, D), \{(D, F), (F, D)\}, (F, F)\}$ . Note that had the duopolist behaved in an identical fashion independent of whether the competitor was dedicated or flexible (i.e., had the two cost thresholds been identical), there would be no asymmetric equilibria, and the solution to a duopoly would have been a relatively straightforward generalization of the problem of a monopolist (the equilibria would have been  $\{(D, D), (F, F)\}$ ). Finally, we wish to emphasize that the stochastic effect is the main driving force of the game since, in its absence, both firms would always invest in dedicated technology.

The market size effect is caused by the differences in the expected market sizes and results in an additional set of equilibria  $\{(D, D), (F, F)\}$  in which the firm mimics the competitor. We note that this

equilibrium is different from the (D, D), (F, F) equilibria arising under the stochastic effect where we know exactly which equilibrium is played, either (D, D) or (F, F). In the additional set of equilibria arising due to the market size effect, both (D, D) and (F, F) are possible.

The cost effect arises from asymmetries in the costs of the two firms. As detailed in Section 4.2.4, asymmetry in the costs of dedicated technology can introduce additional equilibria (D, F) or (F, D). Note again that these equilibria differ from the asymmetric equilibria  $\{(D, F), (F, D)\}$  arising under the stochastic effect alone because, in the latter, both (D, F) and (F, D) are simultaneously possible, whereas the cost effect introduces just one equilibrium – either (D, F) or (F, D) – for a given set of parameters. Finally, asymmetry in the flexible costs does not introduce any new equilibria but may shift the equilibrium of the game.

Given the many possible outcomes, it is somewhat hard to illustrate all possible equilibria graphically. However, the exact equilibrium of the technology game for a given set of parameters can be obtained from delineating the specific cost thresholds of firms i and j, as in Proposition 3.

# 5 Discussion of assumptions and limitations

Our analysis relies on several assumptions whose impact on our results we discuss in this section. These assumptions are: risk-neutral decision makers, strictly positive production quantities for both products (Assumption A-1), production clearance (Assumption A-2) and investment in only one type of technology.

**Risk neutrality.** Although throughout the paper we assume that firms are risk-neutral, in our model firms behave in a risk-seeking manner because profits increase in demand uncertainty. However, in practice high uncertainty in payoffs may be undesirable, leading the firms to make capacity investment decisions conservatively. Such behavior can be incorporated in our model by introducing an appropriate utility function, e.g.,

$$U = \max_{K} E(\pi_i) - r\left(Var(\pi_i)\right),\tag{4}$$

where the definition of  $\pi_i$  depends on the subgame and parameter r represents the measure of the company's risk aversion. This certainty equivalence utility function is quite standard in economics, finance and management (see Gollier 2001, page 20). Even with this simple utility function the analysis of the technology game is quite complex, so we obtain only the optimal capacity decision and utility in the dedicated subgame with  $\beta = 0$ . We use this result to comment on the likely impact of risk aversion on the outcome of the entire game.

**Proposition 6** For a firm with utility function (4) in the dedicated subgame with  $\beta = 0$ , (i) SPNE in (total) capacity is  $K^d = (\mu_1 + \mu_2 - 4c_i + 2c_j) / (3 + 2r\sigma_y^2)$ , and

#### (ii) The expected utility at equilibrium is

$$U^{d} = \sum_{y=1}^{2} \left( 1 + r\sigma_{y}^{2} \right) \left( \frac{\mu_{y} - 2c_{i} + c_{j}}{3 + 2r\sigma_{y}^{2}} \right)^{2}.$$

If we let r = 0, Proposition 6 simplifies to the familiar result in Proposition 1. We note that, as the variance of demand intercepts increases, the capacity investment falls, which is consistent with our intuition regarding the risk-averse decision maker. Likewise, as demand uncertainty increases, the optimal utility decreases. Unfortunately, similar results are hard to obtain for the flexible or the mixed subgames because the optimal decision depends in a nontrivial way on the third and higher moments of the probability distribution of demand intercepts. We conjecture that, similar to the result of Proposition 6, both the optimal capacity investment and the deterministic component of the optimal utility will decrease in the variance of the demand intercepts. However, the stochastic component will remain unaltered (which can be seen from the proof of Proposition 6). Hence, for the flexible firm, risk aversion leads to two countervailing effects: on the one hand, the firm prefers higher demand uncertainty in order to make use of flexible capacity (the stochastic effect), while on the other hand, the firm prefers lower demand uncertainty due to risk aversion. Which of the two effects dominates depends on the degree of risk aversion r. We further expect that higher r will decrease cost thresholds governing the equilibria of the game, thus expanding areas in which the dedicated technology is preferred and reducing areas in which the flexible technology is preferred.

Strictly positive production quantities (Assumption A-1). Throughout the paper we rely on the assumption that the flexible firm always produces nonzero quantities of both products for any realization of demand intercepts. Clearly, when one product is much more profitable than the other (due to a high difference in demand intercept realizations), the firm may wish to manufacture just one product, transferring all capacity from the smaller market to the larger market. Assumption A-1 neglects this rather extreme use of flexibility, and we believe that this assumption is quite innocuous. First, this is a plausible assumption from a practical perspective: it is highly unlikely that a manufacturing capacity able to produce two products would be built without a high-enough level of certainty that both products would actually be produced later on. Second, note that we do allow the production quantity of the undesirable product to be arbitrarily small (infinitesimal), but not zero. If the optimal production quantity for a product were zero, forcing it to be very small instead is unlikely to affect the outcome of the game significantly, because the contribution of this product to the firm's profit will be very small. Since this assumption applies only to the flexible firm, we would expect an insignificant decrease in the flexible firm's profit compared to the optimal solution. Hence, we do not expect the findings of the paper to change significantly.

Market clearance (Assumption A-2). By forcing the firms to produce at capacity level for any demand intercept realizations, we introduce suboptimal behavior for both dedicated and flexible technologies, because firms might be forced to charge negative prices in the market with very low demand intercept realizations. See Chod and Rudi (2005), Deneckere et al. (1997) and Anand and Girotra (2003) for examples of similar assumptions. From a practical perspective, firms often find it difficult to produce below capacity in view of large fixed costs associated with production ramp-up and commitments to suppliers (though these issues are not modeled here explicitly). For instance, as Mackintosh (2003) points out, car makers have been forced to slash prices to keep lines running as models fall out of favor with the public, rather than reduce production. For the same practical reason, negative prices are tenable in many situations: for example, it is well known that automobile manufacturers often sell cars below cost to maintain high capacity utilization (see Mackintosh 2003, Holweg and Pil 2004).

In the absence of clearance, Chod and Rudi (2005) demonstrate that the optimal capacity investment increases in demand uncertainty, an effect that we lose due to the simultaneous application of Assumptions A-1 and A-2 (for small enough demand uncertainty Assumption A-2 alone would lead to this outcome, because the firm would find it beneficial always to produce both products, but for large demand uncertainty both assumptions are needed). We acknowledge this as a limitation of our analysis. However, since our main goal is to compare profits in different technology subgames and not capacities, we believe that this effect is of minor consequence to our analysis. Note that clearance affects the dedicated firm more than the flexible firm, because the flexible firm can always reallocate capacity ex-post the demand realization to the market with high demand. (At worst, the flexible firm can mimic the dedicated firm in terms of capacity allocation.) As a result, the clearance assumption favors flexibility over dedicated technology, but we believe that this effect is relatively minor, as Chod and Rudi (2005) demonstrate numerically. Nevertheless, even in the presence of clearance we demonstrate throughout the paper that flexibility has limited benefits as a response to competition. We expect this finding to strengthen without the clearance assumption.

More important, the clearance assumption preserves the nature of the four effects – stochastic, market size, product substitutability and cost – which are the main drivers behind the equilibrium outcome of the technology game. The stochastic and the market size effects are driven by the convexity of the profit function in the random demand intercepts, which is still the case even without imposing Assumption A-2, though these effects might be weaker. The product substitutability effect amplifies these two effects and is independent of the clearance assumption. The cost effect is mainly driven by a cost differential and hence is independent of the clearance assumption. Lastly, Chod and Rudi (2005) find numerically that Assumptions A-1 and A-2 generally yield solutions that are very close to optimal, and in Goyal and Netessine (2006) we show that it is possible to develop appropriate analytical restrictions on the domain of the probability distributions of demand intercepts such that these assumptions hold naturally.

**Investment in one type of capacity.** Firms often invest simultaneously in dedicated and flexible manufacturing capacities. For example, cheaper dedicated capacity can be used to cover the proportion of demand that is relatively certain, whereas the more expensive flexible capacity can be utilized to cover the

highly uncertain portion of demand. Such a strategy may be preferred to investing in just one capacity type unless there are significant additional fixed costs. If we were to allow the firm to invest in two types of capacities, the resulting problem would differ significantly from the setting we focus on, because the ability to invest in both technologies renders the technology game obsolete. Instead, one could compare the proportions of flexible and dedicated capacity that the firm builds. This is clearly a different problem that is likely to result in different insights and hence merits a separate study. See also Bish and Wang (2004) for analytical difficulties associated with the analysis of a simultaneous investment in dedicated and flexible capacity even in a monopoly setting.

# 6 Conclusion

In this paper we consider the technology choice and capacity investment of two firms facing stochastic price-dependent demand in a competitive market. Each firm makes three decisions: choice of technology, choice of capacity, and choice of production quantity. Hence, we cover all three levels of firms' decisions: strategic, tactical and operational. After introducing appropriate simplifying assumptions, we solve the entire game in closed form. We develop the best responses of the firms in the technology game and compare the best response of a duopolist with the behavior of a monopolist. We show that insights into the outcome of the technology game are confounded by the presence of four distinct effects: the stochastic effect, the market size effect, the product substitutability effect and the cost effect. We discuss each effect separately and delineate its impact upon the equilibria of the technology game. We show that all four equilibria – (F, F), (D, D), (D, F) and (F, D) – could arise depending on the specific values of the problem parameters. Specifically, asymmetric equilibria can arise even if the two firms are completely symmetric, and hence, different technologies might coexist in the market.

Our findings provide systematic answers to questions regarding the value of flexibility as a competitive weapon. Anecdotal evidence from the popular press suggests that flexibility is universally "good" in a competitive environment. Our results, however, point out that a variety of other equilibrium outcomes are also possible, including some in which both firms invest only in dedicated capacity or in which two different production technologies may coexist. Overall, we show that flexible technology is not a panacea for all evils – there are conditions under which dedicated technology emerges in equilibrium, and flexibility is not a universal best response to competition. Our results lead to several hypotheses that could be tested empirically. Namely, flexibility should be favored under high demand uncertainty, low demand correlation, low total market size, and low market size differential, and its adoption should also depend on the competitor's decision. We pursue an empirical study separately in Goyal et al. (2006).

Our results come with several limitations. In our work we do not endogenize economies of scope. Further, we take a restrictive view of flexibility as product flexibility only and not, for example, volume flexibility. By incorporating a reduction in lead times, economies of scope, and the advantage for new product development, the benefits of flexibility could definitely be increased further. It would also be interesting to study flexibility as an entry determined tool under stochastic demand. Moreover, allowing the flexible firm to enter another market, i.e., allowing it to manufacture a third product, would influence the choice of technology for the rival firm. Many firms have focused on the role of flexible technology in developing prototypes, thereby drastically shortening the time to market for a newly developed product. However, there are few analytic models to this effect. This should prove to be an interesting problem for further research.

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# Appendix

## **Proof of Proposition 1**

**Proof.** For the (D,D) subgame and under Assumption A-1, the expected profit for firm i in the capacity game is:

$$\Pi_{i}^{d} = \max_{K_{1i}^{d}, K_{2i}^{d}} (\mu_{1} - K_{1i}^{d} - K_{1j}^{d} - \beta(K_{2i}^{d} + K_{2j}^{d})) K_{1i}^{d} + (\mu_{2} - K_{2i}^{d} - K_{2j}^{d} - \beta(K_{1i}^{d} + K_{1j}^{d})) K_{2i}^{d} - c_{i} \left(K_{1i}^{d} + K_{2i}^{d}\right).$$

After obtaining the first-order conditions for capacities and solving simultaneously for both players we derive the expressions for the optimal capacity choices. Optimal profit expressions follow.

For the (F,F) subgame and under Assumptions A-1 and A-2, the optimal profit of firm i in the production game can be written as

$$\pi_{i}^{f} = \max_{q_{1i}^{f}, q_{2i}^{f}} \sum_{y=1}^{2} \left[ \left( A_{y} - \left( q_{yi}^{f} + q_{yj}^{f} \right) - \beta \left( q_{(3-y)i}^{f} + q_{(3-y)j}^{f} \right) \right) q_{yi}^{f} \right],$$
  
s.t.  $q_{1i}^{f} + q_{2i}^{f} = K_{fi}^{f}, q_{yi}^{f} > 0, \ y = 1, 2.$ 

After obtaining the first-order conditions for production quantities and solving simultaneously for both players we obtain the equilibrium production quantities,  $\hat{q}_{yi}^{f}$ . The expected profit in the capacity game for firm *i* is

$$\Pi_{i}^{f} = \max_{K_{fi}^{i}} E_{A_{y}} \sum_{y=1}^{2} \left[ \left( A_{y} - \left( \hat{q}_{yi}^{f} + \hat{q}_{yj}^{f} \right) - \beta \left( \hat{q}_{(3-y)i}^{f} + \hat{q}_{(3-y)j}^{f} \right) \right) \hat{q}_{yi}^{f} \right] - c_{fi} K_{fi}^{f}.$$

After obtaining the first-order conditions for capacities and solving simultaneously for both players we derive the expressions for the optimal capacity choices. Optimal profit expressions follow.

For the (D,F) and (F,D) subgames and under Assumptions A-1 and A-2, the optimal profit of flexible firm *i* in the production game is

$$\pi_f^m = \max_{q_{1f}^m, q_{2f}^m} \sum_{y=1}^2 \left[ \left( A_y - \left( q_{yf}^m + K_y^m \right) - \beta \left( q_{(3-y)f}^m + K_{(3-y)}^m \right) \right) q_{yf}^m \right],$$
  
s.t.  $q_{1f}^m + q_{2f}^m = K_f^m, q_{yf}^m > 0, \ y = 1, 2,$ 

whereas the optimal profit of the dedicated firm is

$$\pi_d^m = \sum_{y=1}^2 \left[ \left( A_y - \left( q_{yf}^m + K_y^m \right) - \beta \left( q_{(3-y)f}^m + K_{(3-y)}^m \right) \right) K_y^m \right].$$

The optimal production quantities for the flexible firm can be obtained from the first-order conditions. The expressions for the optimal expected profit in the capacity game for the flexible firm and the dedicated firm can be obtained similar to the (F,F) and (D,D) subgames. After obtaining the first-order conditions for capacities and solving simultaneously for both players we derive the expressions for the optimal capacity choices. Optimal profit expressions follow.

### **Proof of Proposition 2**

**Proof.** Part (i) is analogous to the proof of Proposition 1. For part (ii), we define  $\delta_m^{f-d} \equiv \prod_f^M - \prod_d^M$  as the incremental profit a monopolist makes when investing in flexible technology over dedicated technology. After substituting the profit expressions from part (i) and simplifying, we obtain  $\delta_m^{f-d}$  as a quadratic function of  $c_f$ . The smaller root of this function (because the capacity investment is negative at the higher root) is the required threshold.

#### **Proof of Proposition 3**

**Proof.** For part (i), we define  $\delta_i^{f-d} \equiv \prod_{if}^m - \prod_i^d$  as the incremental gain that firm *i* makes by being

flexible when firm j is dedicated. Substituting the expressions for the profits from Table 2 and simplifying, we obtain  $\delta_i^{f-d}$  as a quadratic function of  $c_{fi}$ . The smaller root of this function (because the capacity investment is negative at the higher root) is the required threshold. Part (*ii*) is shown analogously after defining  $\delta_i^{f-f} \equiv \prod_i^f - \prod_{id}^m$ . Parts (*iii*) and (*iv*) follow directly from the expressions of the thresholds.

## **Proof of Proposition 4**

**Proof.** We let  $\Delta^2 = (\mu_1 - \mu_2)^2$ . It is straightforward to show that  $\partial \bar{c}_{fi} / \partial \Delta^2$ ,  $\partial \underline{c}_{fi} / \partial \Delta^2 \leq 0$ . Hence, both thresholds decrease as  $\Delta$  increases. The inequality  $|\partial \bar{c}_f / \partial \Delta^2| \geq |\partial \underline{c}_f / \partial \Delta^2|$  follows after some algebraic manipulations and by noting that  $c_{fj} \geq c_j$  and  $|\beta| < 1$ .

## **Proof of Proposition 5**

**Proof.** Results (i) follow directly from the expressions of cost thresholds. To establish  $\partial \bar{c}_{fj}/\partial c_i \leq 0$ , note that firm j's incremental profit if it invests in flexibility (given that firm i is dedicated) is

$$\delta_j^{f-d} = \frac{(\mu_1 + \mu_2 + 2c_i - 4c_{fj})^2}{18(1+\beta)} + \frac{(\mu_1 - \mu_2)^2}{32(1-\beta)} + \frac{\sigma_T^2}{8(1-\beta)} - \frac{(\mu_1 + \mu_2 - 4c_j + 2c_i)^2}{18(1+\beta)} - \frac{(\mu_1 - \mu_2)^2}{18(1-\beta)}.$$

After differentiating the above expression w.r.t.  $c_i$  and noting that  $c_{fj} \ge c_j$ , we obtain  $\partial \delta_j^{f-d} / \partial c_i \le 0$ . Since firm j makes incrementally less by being flexible as  $c_i$  increases, it follows that  $\partial \bar{c}_{fj} / \partial c_i \le 0$ . Similarly, to demonstrate  $\partial \underline{c}_{fj} / \partial c_{fi} \le 0$ , define  $\delta_j^{f-f}$  as firm j's incremental profit if it invests in flexibility, given that firm i is flexible. Then, we can show that  $\partial \delta_j^{f-f} / \partial c_{fi} \le 0$ . Hence,  $\partial \underline{c}_{fj} / \partial c_{fi} \le 0$ .

#### **Proof of Proposition 6**

**Proof.** The variance of the profit in the production game is  $Var\left(\pi_{i}^{d}\right) = E\left(\pi_{i}^{d}\right)^{2} - \left(E\pi_{i}^{d}\right)^{2}$ . It can be shown that  $Var\left(\pi_{i}^{d}\right) = \sum_{y=1}^{2} \sigma_{y}^{2} \left(K_{yi}^{d}\right)^{2}$ . Hence, the firm maximizes the following utility function:

$$U_{i}^{d} = \max_{K_{1i}^{d}, K_{2i}^{d}} \left( \sum_{y=1}^{2} \left( \left( \mu_{y} - \left( K_{yi}^{d} + K_{yj}^{d} \right) \right) K_{yi}^{d} - r\sigma_{y}^{2} \left( K_{yi}^{d} \right)^{2} - c_{i} \left( K_{yi}^{d} \right) \right) \right).$$

This problem is clearly concave in the decision variables. After finding the first-order conditions and simplifying, we obtain the result.  $\blacksquare$