HOW TO PROJECT PATIENT PERSISTENCY

Ka Lok Lee, Peter Fader, and Bruce Hardie

PREVIEW

Pharmaceutical companies face the problem of how to project the persistency patterns of patients who are taking their manufactured medications – i.e., how to determine the percentage of patients who will continue to refill a given prescription on a timely basis. The authors have developed a probability model with a well-grounded story for the dropout process. The model, which can be implemented in a simple Excel spreadsheet, provides remarkably accurate forecasts as well as other useful diagnostics about patient persistency.

KEY POINTS

- Forecasting patient persistency in taking medications is an example of the general forecasting challenge of predicting consumer retention rates.
- We describe a probability mixture model to capture and project persistency patterns over time.
- Extensions to the model allow us to address persistency differences among demographic groups.

INTRODUCTION

Patient persistency to prescribed medications, defined as the fraction of patients who continue to refill a given prescription on a timely basis, has become a serious issue, garnering significant attention from pharmaceutical firms and public health officials. A recent article in *DTC Perspectives* (Rubin, 2006) suggests that low levels of persistency lead to 125,000 deaths per year and cost the U.S. health care system $100 billion annually. Another article claims that, next to launching a new blockbuster medication, improving persistency is the primary marketing strategy on which pharmaceutical firms should focus, since it can generate a substantial return on investment (Meadows, 2006).

While this issue might seem unique to the pharmaceutical industry, a similar problem plagues managers in other industries, such as telecommunications, financial services, and magazine subscriptions, where it is known as customer retention (Fader & Hardie, 2007).

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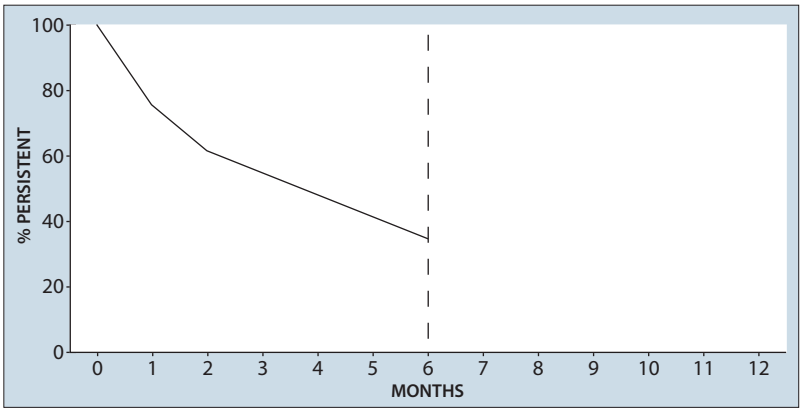
Peter S. Fader is the Frances and Pei-Yuan Chia Professor of Marketing at the Wharton School of the University of Pennsylvania. He enjoys building simple probability models that are easy to implement and broadly applicable across multiple managerial domains. The "persistency" model presented in this paper is an excellent example of both of these desirable characteristics. Another model of persistency is Len Tashman, the editor of this journal, who was extraordinarily supportive and helpful in bringing this article to fruition. Pete and his co-authors sincerely appreciate his encouragement and the opportunity to reach out to the readers of *Foresight*.

Bruce Hardie is an Associate Professor of Marketing at London Business School. He holds MA and PhD degrees from the University of Pennsylvania and B.Com and M.Com degrees from the University of Auckland (New Zealand). His primary research interest lies in the development of data-based models to support marketing analysts and decision makers. Bruce's research has appeared in various marketing, operations research, and statistics journals.

Figure 1. An Illustrative Persistency Curve



A PERSISTENCY CURVE

Persistency patterns can be visualized using a *persistency curve*, a typical example of which is given in Figure 1.

A persistency curve tells us what percentage of a cohort of patients (those who started the medication therapy at the same time) remain "persistent" (i.e., have not dropped out of therapy) over time. Here are three basic characteristics of a persistency curve:

i) It starts out at 100% and works its way down towards 0% as time increases.
ii) It is non-increasing.
iii) It tends to decrease at a decreasing rate over time.

The persistency curve given in Figure 1 tracks the behavior of 1000 new patients for a chronic hypertensive medication. The associated raw data are presented in Table 1. Such persistency data are typically collected by market research firms, who collate patient-level prescription data from retail pharmacy chains.

Table 1. Persistency Data for a Chronic Hypertensive Medication

MONTH	# PATIENTS	% PERSISTENT
0	1000	100.0%
1	758	75.8%
2	617	61.7%
3	554	55.4%
4	483	48.3%
5	417	41.7%
6	350	35.0%

From a forecasting perspective, the key question is, having observed the patient dropout pattern from month 1 to month 6, how many more dropouts are going to occur over the succeeding months – for example, months 7–12?

A natural starting point would be to fit a simple function of time to these data. For example, we could project that the persistency rate declines by a constant percentage per month. However, Fader and Hardie (2007) showed that while this type of model can fit the observed data reasonably well, it typically breaks down dramatically in the forecast period. Not only is it unable to provide an accurate forecast, it may also violate the characteristics of a persistency curve as mentioned above. Rather than simple curve-fitting, therefore, we need to create a model with a proper "story" of patient-level behavior.

Our story of the dropout process leads to a simple probability model that can be used to forecast the pattern of patient persistency. This model was originally raised in the context of customer retention (Fader & Hardie, 2007), but we show that it provides remarkably accurate forecasts and other useful diagnostics about patient persistency.

A PROBABILITY MODEL FOR PATIENT PERSISTENCY

Consider the following story of a typical patient as he decides whether or not to refill a prescribed medication:

i) At the end of each refill cycle (e.g., month), the patient flips a coin: if it comes up "heads" he returns to a pharmacy for a refill of his medication; if "tails" he stops using this medication.
ii) For any given patient, the probability of heads or tails does not change over time.
iii) The coin isn't necessarily a fair one; the probability of heads or tails isn't necessarily 50%.
iv) Each patient has his own coin, so the probability of heads or tails varies across patients.

This story is not intended as a strict description of the decision process. By no means are we suggesting that any patient will really make his health care decisions with a simple flip of a coin. There are many factors that drive the decision to refill or drop the prescription, but these factors are largely unobservable to the researcher. We therefore treat the behavior as if it is random. The probability of tails describes the propensity to stop refilling the medication (i.e., drop out).

Our approach to model building is to start with the simplest set of reasonable assumptions, adding richer details (and potential complications) only if the simpler model does not work well.

This story recognizes that persistency is an individual-level decision, with a sizeable element seen as random to an outside observer, and that the propensity to maintain or drop the prescription will likely vary from patient to patient. Some readers may question assumption (ii), the stationarity of the coin, arguing that the individual-level likelihood of dropping out may change over time. However, our approach to model building is to start with the simplest set of reasonable assumptions, adding richer details (and potential complications) only if the simpler model does not work well. It is easy to introduce an element of individual-level time-dependence to this model, but as we will see shortly, our basic model generates a very accurate forecast and therefore serves our purposes well.

As noted earlier, our coin-flipping model is adapted from the customer-retention forecasting work of Fader & Hardie (2007). We refer the interested reader to that paper for complete details of the model and present only the key features here.

[1] The point in time at which a patient stops refilling the medication (drops out) follows a probability distribution called the (shifted) *geometric distribution.* Suppose the probability of dropping out at the end of each month is denoted by θ (theta). According to the (shifted) geometric distribution, the probability that dropout occurs at the end of the *first* month is θ; the probability that it occurs at the end of the *second* month is θ(1–θ); the probability that it occurs at the end of the *third* month is $\theta(1-\theta)^2$, and so forth.

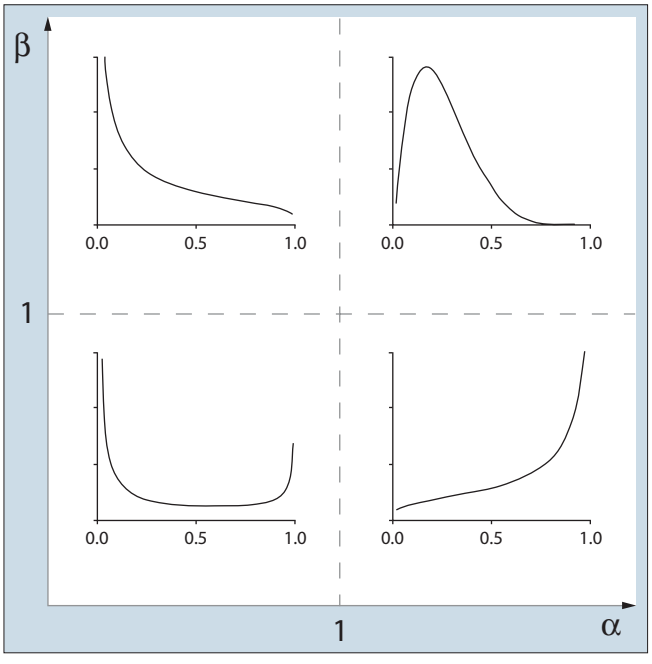
Let us consider a numeric example. Suppose a patient's dropout probability is θ = 0.4. The probability of this patient dropping out by the end of the second month is the sum of the probability that he drops out at the end of the first month (0.4) and the probability that, as a first-month "survivor," he drops out at the end of the second month (0.4*(1–0.4)), which adds up to 0.64.

[2] θ will vary from person to person. To account for this heterogeneity, we assume that θ follows a *beta distribution*, which is an extremely flexible two-parameter (α and ß) probability distribution that can capture many different patterns of between-patient heterogeneity in the dropout propensities. Different values of α and ß can lead to different shapes for the beta distribution, the general patterns of which are shown in Figure 2.

If both parameters are small (<1), as in the lower-left panel, then the distribution of dropout probabilities is U-shaped, indicating that the dropout decision is highly polarized across patients and the patients are either very persistent or very quick to drop out of therapy. If both parameters are relatively large (α, ß > 1), as in the upper-right panel, then there is an interior mode, implying that the dropout probabilities are clustered around a peak value that lies between 0 and 1. Likewise, the distribution of probabilities can be J-shaped, as in the lower-right panel (i.e., most patients have high dropout propensities) or reverse-J-shaped, as in the upper-left panel (i.e., most patients have low dropout propensities). It is not essential for the reader to remember these cases, but these parameters can offer useful diagnostics to help the manager understand the degree (and nature) of heterogeneity in dropout probabilities across patients.

[3] Combining assumptions 1 and 2 gives us a probability mixture model called the (shifted) beta-geometric (sBG)

Figure 2. The Four General Shapes of the Beta Distribution for Different Values of α and β



distribution. The model can be used to calculate the probability that a randomly chosen patient drops out of therapy at the end of each month. More formally, the probability that a randomly chosen patient drops out of therapy at the end of month t can be computed using the following forward-recursion formula:

$$P(T = t) = \frac{\alpha}{\alpha + \beta} \qquad t = 1$$

$$P(T = t) = \frac{\beta + t - 2}{\alpha + \beta + t - 1} \; P(T = t - 1) \quad t = 2,3,...$$

Given the observed persistency data, it is a straightforward exercise to estimate the two model parameters. Appendix 2 of Fader and Hardie (2007) shows how this can be done using Excel. We can then project the persistency curve into the future.

For the persistency data presented in Table 1, the estimates of the two sBG model parameters are $\alpha = 1.08$ and $\beta = 3.67$. The resulting model-based estimate of the persistency curve for months 1-12 is plotted along with the actual persistency curve in Figure 3.

Figure 3 shows how closely the sBG model-based estimate matches with the actual persistency curve. The model overestimates the month-12 persistency level by only two percentage points.

In addition to this forecast, we are able to make inferences about how the dropout probabilities vary across the patient population. A plot of the estimated beta distribution is presented in Figure 4.

The distribution is effectively reverse J-shaped, suggesting that there is a high degree of heterogeneity across the population. While a relatively small number of patients have high dropout probabilities, most of the population will be reasonably persistent in their prescription-refill behavior over time. It is interesting to note that this is the same shape observed for the two datasets modeled by Fader and Hardie (2007). In numerous other applications of this model, we have seen that this reverse J-shape is far and away the most common characterization of consumer heterogeneity across a variety of domains.

DISCUSSION AND EXTENSIONS

We have shown that a simple probability model – even one borrowed from a domain seemingly unrelated to pharmaceuticals – not only does a good job of capturing patient persistency but also offers a useful story about it. We have presented the (shifted) beta-geometric distribution as a simple but effective model for patient persistency behavior and demonstrated that it can provide a good forecast and useful diagnostics about the patient population.

The sBG model can be extended for additional analyses of patient persistency. For example, we can use the model to make comparisons of patient persistency across different cohorts of patients and medications in the same category. We can also use it to evaluate the effects of external factors (e.g. direct-to-patient advertising) on persistency. These managerial questions can be addressed with the following extensions:

i) Fit separate sBG models to different cohorts of patients, such as male vs. female, patients in a direct-to-physician campaign vs. patients not in such a campaign, and compare the means and distributions of the dropout probabilities.

Figure 3. Comparing the Actual Persistency Curve with the Model-based Estimate

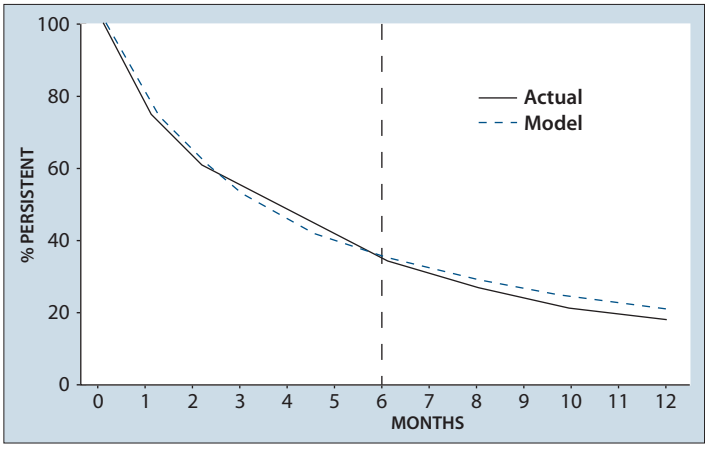
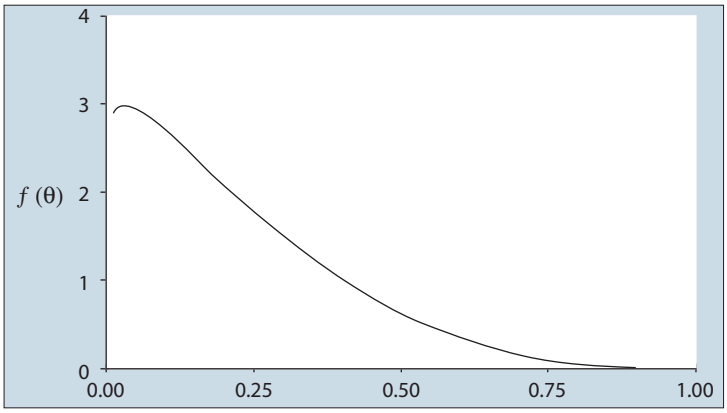


Figure 4. Distribution of Dropout Probabilities Across the Patient Population



ii) Make α and β functions of cohort characteristics and patient descriptor variables such as age and gender (Rao & Steckel, 1995).

Such extensions can offer useful insights into how marketing efforts (e.g., patient targeting) can be performed in a more effective manner.

In conclusion, as the topic of patient persistency (and the broader issue of customer retention) gains more and more attention from managers in the pharmaceutical industry and elsewhere, we highly recommend the use of the sBG model as a simple starting point to better understand and forecast persistency and retention patterns over time.

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