# An Incentive Effect of Multiple Sourcing*

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Abstract: We consider a supply chain with one manufacturer who assembles an end-product consisting of two components (A and B), where each component is purchased from multiple outside suppliers. The manufacturer's decision as to the contract to offer each supplier is complicated by two factors: (1) each potential supplier is self-interested and privately-informed as to his marginal cost of production; and (2) any A parts delivered in excess of B parts (or vice versa) cannot be inventoried but must be disposed of at no value. Thus, the manufacturer must choose the contract to offer each so as to coordinate or balance their production, while taking into consideration their self-interested behavior. We demonstrate that increasing the number of suppliers mitigates (but does not entirely eliminate) the production distortions arising from these two factors. Hence, we show that increasing the supplier base can have efficiency enhancing effects that have not been previously described in the literature.

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## 1. Introduction

Two major trends in manufacturing within the last two decades have been: the increased use of outsourcing of parts and subassemblies and the reduction in the number of first-tier suppliers to whom these parts and subassemblies are outsourced (see, for example, (McMillan, 1990)). The efficiency effect of the number of first-tier suppliers is usually analyzed in terms of manufacturing (dis)economies of scale, risk of supply disruption and coordination costs.[2] While these three effects have been studied before, we demonstrate that there is another, incentive-related, efficiency effect associated with the number of suppliers which has not been studied in the literature.[3] This incentive-related efficiency effect arises when the suppliers are privately-informed and self-interested and there is a need to balance their production (as is the case in assembly systems).[4]

To demonstrate this incentive efficiency effect, we model a simple assembly process with a manufacturer (referred to as the buyer and she) whose end-product is made up of two components (parts A and B). The buyer outsources the production of both parts to suppliers (each referred to as him) and merely assembles the end-product from the parts delivered. Each supplier's cost to manufacture his parts is uncertain at the time of contracting. Before each supplier produces his parts, he learns his cost realization and communicates it to the buyer. The latter then informs each supplier of the desired production schedule. The incentive efficiency effect associated with the number of suppliers arises from the following two assumptions. First, the suppliers are privately-informed about their cost realizations and are self-interested; each acts so as to maximize his own expected profit. As a result, the privately-informed suppliers will not necessarily truthfully communicate their cost observations to the buyer (this is referred to as the adverse selection or supplier-incentive problem). Second, each of the buyer's end-products requires one part A and one part B and, because of spoilage, fashion concerns, or other constraints, the buyer cannot inventory any unused parts for use at some later date. Thus the

[2] Coordination costs typically include the cost of communicating with and keeping track of the activities of suppliers.
[3] For the effects of supply chain disruptions see, for example, (Reitman, 1997), (Latour, 2001) and (Hendricks and Singhal, 2003). For a discussion of coordination costs see, for example, (McMillan, 1990).
[4] See (Baiman and Rajan, 2002) for a survey of the research literature which addresses the incentive implications of other sourcing decisions.

buyer has to balance or coordinate the suppliers' production schedules because any A parts delivered in excess of B parts (or vice versa) are valueless and must be discarded (this is referred to as the production-balancing problem). The presence of these two assumptions induces the buyer to *distort* the optimal production schedules offered to the suppliers (relative to the optimal production schedules in the absence of a supplier-incentive problem). This production distortion reduces the buyer's efficiency and expected profit. We show that increasing the number of suppliers *reduces* the relative size of each supplier's production distortion, thereby increasing efficiency.[5] The reason for our result is that increasing the number of suppliers reduces the effect that any one supplier can have by misrepresenting his private information, thereby reducing the production distortion required to elicit the truth and improving coordination. This result relies on both of our earlier stated assumptions. With no supplier-incentive problem there is no need to distort production in order to truthfully elicit each supplier's cost realization. With no production-balancing problem, the optimal contracts treat each supplier as independent; hence there is no relative production distortion reduction effect associated with the number of suppliers.

The paper is organized as follows. Section 2 reviews the relevant literature. In section 3 we discuss in more detail the basic assumptions underlying the model. We also solve for the optimal supplier contracts and supplier production schedules assuming the production-balancing problem but assuming away the supplier-incentive problem. That is, we assume that the suppliers honestly report their information and implement whatever production schedules the buyer gives them. This is referred to as the First-Best solution and provides a benchmark for our subsequent analysis. We need this benchmark because, even without a supplier-incentive problem, varying the number of suppliers affects the optimal production. In section 4 we derive the optimal supplier contracts and production schedules when there is the supplier-incentive problem (this is referred to as the Second-Best case). We then compare the optimal solutions derived in sections 3 and 4 to isolate the incentive-efficiency effect of the number of suppliers. Section 5 provides a conclusion.

[5] Increasing the number of suppliers can also lead to improved contracting via relative performance evaluation when the agents are subject to correlated shocks (see (Holmstrom, 1982)). This effect does not arise in our model because our suppliers are subject to uncorrelated shocks.

## 2. Literature Review

The present paper lies at the intersection of the procurement/assembly literature in supply chain management and the contracting/incentives literature in economics/accounting. In the assembly literature, as in our paper, the manufacturer (assembler) constructs the end-product from parts provided by suppliers. One subset of this literature looks at the buyer's problem of coordinating the suppliers' production schedules in a centralized setting (i.e., there are no incentive problems in motivating the suppliers – see, for example, (Song et al., 2000) and citations therein). More closely related to our work is the subset of the literature that looks at the buyer's problem of coordinating the suppliers within a decentralized setting – the buyer and suppliers make their respective decisions in their own best interests based on their contracts. In much of this literature, unlike in the present paper, there is no underlying incentive problem – any conflict between the interests of suppliers and the buyer is caused solely by the assumed contracts between them (e.g., (Wang and Gerchak, 2003) and (Bernstein and DeCroix, 2003)). That is, it is the use of non-optimal contracts which create the supplier-incentive problem. Closest to our work is (Gurnani and Gerchak, 2003) which does analyze a setting in which there is an underlying incentive problem involving the suppliers. In that paper each supplier's actions are subject to moral hazard and the paper analyzes the efficiency effects of two different (non-optimal) contracts between the buyer and suppliers. In contrast, we study a decentralized decision situation in which each supplier is privately informed about his cost of production (i.e., an adverse selection problem) and analyze the efficiency effects associated with the optimal (screening) contract.

Much of the procurement literature (see the survey by (Elmaghraby, 2000)) analyzes the allocation, to one or more privately-informed suppliers, of the right to provide a single good or service to the buyer. When, instead, the right is to supply multiple units, it is typically assumed that they are identical and that there are no externalities among units or the suppliers.[6] Our paper is different from this literature in a number of ways. First, while much of this literature restricts

[6] See, for example, (Stole, 1994), (Rob, 1986), (Laffont, 1993), (Riordan and Sappington, 1989), (Seshadri, 1991) and (Klotz, 1995).

itself to an auction approach, we take a more general optimal contracting approach.[7] Second, in our model, the goods being supplied are not identical and are not used independently. In particular the buyer must confront production-balancing issues among her suppliers. The efficiency effect of varying the number of suppliers in the present paper is therefore different from that in the procurement literature.[8]

As noted above, our work also fits within the economics/accounting literature dealing with incentives and organizational design in that an assembly system can be interpreted as an organization in which a principal combines inputs from subordinate departments. (Demski and Sappington, 1984) and (Ma et al., 1988) study the implicit coordination among the agents that may arise as a result of their contracts. Under our equilibrium concept implicit collusion does not arise. (Harris et al., 1982) studies the internal resource allocation process in which the principal assembles an end-product made up of parts produced by the privately informed divisions. However, it assumes that each division is a sole source for the part it supplies, while our focus is on the value of multi-sourcing.[9] (Ziv, 1993), (Ziv, 2000) and (Balakrishnan et al., 1998) study the effect of a firm's monitoring technology on its organizational design. (Balakrishnan et al., 1998) focuses on different ways of organizing a fixed number of workers (team vs. individual production). Our paper and (Ziv, 1993) and (Ziv, 2000) study the effect of varying the number of workers. There are two major differences in the assumptions which underlie the works of Ziv and the present paper. First, different assumptions are made with respect to production externalities among the workers/suppliers. Second, the models analyzed by Ziv do not incorporate adverse selection on the part of the workers/suppliers, while the model in the present paper does.

While the above incentives literature assumes complete contracting, another subset allows for incomplete contracting.[10] With incomplete contracts, the number of suppliers with whom a buyer chooses to deal, as well as the way in which the buyer organizes them, can affect

[7] (Tunca and Zenios, 2004) study the relative efficiency of procurement auctions versus long-term relational contracting.
[8] (Cachon and Zhang, 2003) studies the effect of creating externalities among the suppliers through competition.
[9] Also, see (Baiman et al., 2004) for an assembly model with multiple parts but single-sourcing of each part.
[10] A contract is incomplete if it does not specify the desired behavior by the contracting parties for each jointly observable future event. When an unspecified future contingency does occur, the parties must renegotiate the contract.

the range of self-enforcing behaviors of both the buyer and the suppliers. For example, see (Bakos and Brynjolfsson, 1993) and (Levin, 1998). The contracts analyzed in the present paper are complete and, therefore, these issues regarding the number of suppliers do not arise. Finally, there is the literature which studies the strategic effects of outsourcing, (see, e.g., (Shy and Stenbacka, 2003) and (Cachon and Harker, 2002). No such strategic effects exist in our model because the buyer sells her units to a competitive market.

## 3. **Assumptions, notation and first-best analysis**

### a. **Assumptions and notation**

Our model consists of a one-period world with one risk-neutral buyer and *2n* risk-neutral suppliers. The buyer manufactures an end-product made from one part A and one part B. Part A is supplied by a set of *n* suppliers (referred to as part A suppliers) and part B is supplied by a non-overlapping set of *n* suppliers (referred to as part B suppliers).[11] The buyer contracts with the suppliers and assembles the end-products from the parts delivered by them. The suppliers' production functions have a random component (that might be due to uncertainty in equipment quality/yield, random disruptions in the production process, reliability of the transportation, currency fluctuations etc.) that is modeled as follows. The cost to supplier $A_i$ (the *i*th supplier of part A) of supplying $x_i^A$ parts is $FC(n)+c_{ij}^A\left(x_i^A\right)^2$, where *FC(n)* is the fixed cost which the supplier must incur in order to produce any units of A and $c_{ij}^A$ is supplier $A_i$'s *j*th cost realization.[12] The same holds true for the part *B* suppliers. We further assume that: *FC(n)* is the same known constant for both the A and B suppliers; the suppliers' cost realizations are drawn from identical but independent probability distributions; there are only two cost realizations, $c_1$ and $c_2$, where $c_1 < c_2 < \infty$ occur with probabilities $0 < p < 1$ and $1 - p$, respectively; and the

[11] The assumption that the number of A and B suppliers is the same can be easily relaxed.

[12] We allow the cost incurred by each supplier to be a function of the number of other suppliers of that part. One reason for this is that an equivalent interpretation of *FC(n)* is that it represents the additional communication/coordination cost which the *buyer* incurs for each supplier.

information about the cost distribution is common knowledge.[13] $c_1$ is referred to as the low-cost realization and $c_2$ is referred to as the high-cost realization.

At the start of the period, no one has yet observed the cost realizations, but everyone shares common beliefs about those cost realizations. The game proceeds as follows:

Step 1. The buyer offers a contract to each supplier which stipulates how many parts each will be asked to produce and how much each will be paid for doing so, depending upon the information subsequently communicated to the buyer by all suppliers.[14]

Step 2. Each supplier privately observes his own cost realization and communicates information about it to the buyer. Each supplier chooses his message so as to maximize his expected profit given the contract offered in Step 1. The suppliers' messages are jointly observable and, hence, contractible.

Step 3. After receiving the suppliers' messages, the buyer communicates to each supplier the number of parts to be delivered and the payment for doing so, consistent with the contract terms negotiated in Step 1.

Step 4. At this point, each supplier has the right to abrogate the contract negotiated in Step 1 and leave the supply chain unless the number of parts ordered and the promised payment at least satisfy his best outside opportunity, assumed to be zero profit. If any supplier does abrogate his contract, the manufacturer cannot renegotiate the contract terms for the remaining suppliers.[15] If the supplier honors the contract he first invests *FC(n)* and then produces the required number of units.

Step 5. Given that the contract satisfies each supplier's outside opportunity, each supplier delivers the number of parts specified by the buyer.

Step 6. The buyer receives the parts and assembles one finished end-product from one part A and one part B and sells each finished end-product for *$R* per unit. If more A than B

[13] We assume that the suppliers are ex ante identical and that there are only two potential cost realizations in order to more clearly focus on the incentive-efficiency effects of the number of suppliers. The qualitative results would still hold even if the A and B suppliers were not ex ante identical and if there were more than two potential cost realizations.

[14] This is referred to as a screening contract in economics literature. In the above setting, this contract is optimal for the manufacturer (see (Kreps, 1990)).

[15] Abrogation of the contract by a supplier is an off-equilibrium event and, thus, will never occur. However, we need this assumption (or equivalently that the manufacturer cannot design the contract to incorporate a two stage process in case a supplier abrogates) in order to have a non-trivial production-balancing problem.

parts are delivered (or vice versa), the excess parts cannot be used and are costlessly disposed.[16]

### b. First-Best Analysis

As stated above, we are interested in studying the incentive-efficiency effect of the number of suppliers. To do so, we must hold constant the other efficiency effects that the number of suppliers might have. For example, varying the number of suppliers also affects optimal production even without the supplier-incentive problem because of its effect on the marginal cost of production. Therefore in order to separate the incentive from the non-incentive efficiency effects of varying the number of suppliers, in this sub-section we derive the solution to the problem described in the previous section, but with the supplier-incentive problem assumed away. This is referred to as the First-Best solution and is comparable to the solution derived in the centralized assembly literature.[17] That is, we first study the case in which each supplier honestly communicates his cost realization to the buyer and produces whatever the buyer asks, as long as doing so leaves him no worse off than his outside opportunity wage of zero expected profit. We restrict our analysis to symmetric solutions.

Assume that the cost realization is $(k, j)$, i.e., there are $k$ low-cost part A suppliers, $n$-$k$ high-cost part A suppliers, $j$ low-cost part B suppliers, and $n$-$j$ high-cost part B suppliers.

Let:

$x_{2kj}^{A}$ = the number of units a part A supplier with a cost realization of $c_2$ is directed to provide given the $(k, j)$ cost realization.

$T_{2kj}^{A}$ = the compensation to a part A supplier for producing $x_{2kj}^{A}$.

The order sizes and compensations for A suppliers with $c_1$ cost realizations and for B suppliers are defined analogously.

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[16] The analysis would be qualitatively the same as long as there is some cost to having an excess of part or B.

[17] See, for example, (Song et al., 2000).

The total number of A parts produced and delivered to the buyer is thus $(n-k)x^A_{2kj} + kx^A_{1kj}$, the total number of B parts produced and delivered to the buyer is $(n-j)x^B_{2kj} + jx^B_{1kj}$, and the total number of end-units which the buyer can produce is $\min\left((n-k)x^A_{2kj} + kx^A_{1kj};(n-j)x^B_{2kj} + jx^B_{1kj}\right)$. The *min(..;...)* function captures the fact that any A parts in excess of B parts (or vice versa) have no value (i.e., the production-balancing problem). This effect arises in many manufacturing/supply chain settings (e.g., JIT production) in which it is costly to have an excess or shortage of any one part. It is the usual objective function in the assembly literature (e.g., (Wang and Gerchak, 2003)). Given that under First-Best the buyer observes the cost realizations before deciding on production, she will choose $\left(x^A_{2kj}, x^A_{1kj}, x^B_{2kj}, x^B_{1kj}\right)$ such that $(n-k)x^A_{2kj} + kx^A_{1kj} = (n-j)x^B_{2kj} + jx^B_{1kj} = \overline{X}_{kj}$, so as to avoid paying for any unused production. Further, given that each supplier's outside opportunity is assumed to be zero, each is reimbursed only for his out-of-pocket production cost $T^A_{2kj} = FC(n) + c_2\left(x^A_{2kj}\right)^2$, $T^A_{1kj} = FC(n) + c_1\left(x^A_{1kj}\right)^2$, etc. The buyer solves for the optimal $\left(x^A_{2kj}, x^A_{1kj}, x^B_{2kj}, x^B_{1kj}\right)$ for each $(k, j)$ realization in two steps:

1. For each $(k, j)$ realization, choose the cost minimizing vector $\left(x^A_{2kj}, x^A_{1kj}, x^B_{2kj}, x^B_{1kj}\right)$ that satisfies $(n-k)x^A_{2kj} + kx^A_{1kj} = (n-j)x^B_{2kj} + jx^B_{1kj} = \overline{X}^F_{kj}$, for some $\overline{X}^F_{kj}$, where the superscript "F" is used to indicate First-Best.
2. For each $(k, j)$ realization, choose the expected profit maximizing $\overline{X}^F_{kj}$.

*Lemma 1: The optimal production orders for each* $(k, j)$ *realization are:*[18]

[18] Lemma 1 is straight-forward and offered without proof. Other proofs are included in the Appendix.

$$
\begin{aligned}
\overline{X}_{kj}^{F} &= \frac{Rn\left(c_1+\left(c_2-c_1\right)k/n\right)\left(c_1+\left(c_2-c_1\right)j/n\right)}{2c_1c_2\left(2c_1+\left(j/n+k/n\right)\left(c_2-c_1\right)\right)} \\
x_{1kj}^{AF} &= \frac{R\left(c_1+\left(c_2-c_1\right)j/n\right)}{2c_1\left(2c_1+\left(j/n+k/n\right)\left(c_2-c_1\right)\right)} \\
x_{2kj}^{AF} &= \frac{R\left(c_1+\left(c_2-c_1\right)j/n\right)}{2c_2\left(2c_1+\left(j/n+k/n\right)\left(c_2-c_1\right)\right)} \\
x_{1kj}^{BF} &= \frac{R\left(c_1+\left(c_2-c_1\right)k/n\right)}{2c_1\left(2c_1+\left(j/n+k/n\right)\left(c_2-c_1\right)\right)} \\
x_{2kj}^{BF} &= \frac{R\left(c_1+\left(c_2-c_1\right)k/n\right)}{2c_2\left(2c_1+\left(j/n+k/n\right)\left(c_2-c_1\right)\right)}
\end{aligned}
\tag{1}
$$

The effect of the production-balancing problem is manifested in (1) by the fact that the required production by each supplier A is a function of the number of low-cost A and B suppliers, $k$ and $j$.[19] The same is true for each B supplier.

Denote the buyer's profit for the cost realization $(k, j)$ by $\pi_{kj}^{F}$ calculated as follows:

$$\pi_{kj}^{F} = R\overline{X}^{F} - \left(\overline{X}^{F}\right)^2 c_1c_2 \frac{2nc_1+\left(k+j\right)\left(c_2-c_1\right)}{\left(nc_1+k\left(c_2-c_1\right)\right)\left(nc_1+j\left(c_2-c_1\right)\right)} - nFC\left(n\right).$$

After substituting in the optimal $\overline{X}_{kj}^{F}$, we obtain

$$\pi_{kj}^{F} = \frac{R^2\left(nc_1+k\left(c_2-c_1\right)\right)\left(nc_1+j\left(c_2-c_1\right)\right)}{4c_1c_2\left(2nc_1+\left(j+k\right)\left(c_2-c_1\right)\right)} - nFC\left(n\right) = \frac{R\overline{X}_{kj}^{F}}{2} - nFC\left(n\right).$$

For each $(k, j)$ realization, total expected profit is linear in total production so that, except for the fixed cost charge, the effect of $n$ on total expected profit and on total expected production is proportional.[20] Let $P\left(k, j, n\right) = p^j\left(1-p\right)^{n-j}\dfrac{n!}{j!\left(n-j\right)!}p^k\left(1-p\right)^{n-k}\dfrac{n!}{k!\left(n-k\right)!}$ represent the probability of the *(k,j)* cost realization given that there are $n$ suppliers for each part. The buyer's total expected profit is then:

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[19] In the absence of the production-balancing problem, each individual supplier's production would be independent of the number of the other suppliers and their cost realizations. See the later discussion.

[20] This effect is due to linearity of the revenue function.

$$\frac{R}{2}\sum_{j=0}^{n}\sum_{k=0}^{n}\bar{X}_{kj}^{F}P\left(k,j,n\right)-nFC\left(n\right). \qquad (2)$$

## 4. Incentive problem

### a. Second Best analysis

We next analyze the Second-Best case, in which the buyer faces self-interested suppliers and must induce them to reveal their privately-observed cost realizations and follow the production schedule given to them. Both the First-Best and Second-Best formulations incorporate the production-balancing problem. Thus, the only difference between the First-Best solution derived in section 3 and the Second-Best solution derived in this section is the existence of the supplier-incentive problem. Hence, any difference between the solutions, and any difference in the effect of the number of suppliers on those solutions, is due totally to the supplier-incentive problem.

Given the Revelation Principle (see (Kreps, 1990)), we can, without loss of generality, restrict the space of contracts to those which induce each supplier to honestly reveal his cost realization to the buyer and implement the specified production schedule.[21] The Second-Best problem can then be stated as:

[21] The usual formulation of the Truth-Telling constraint uses the Bayesian Nash equilibrium concept which states that each supplier's expected profit, *given his cost realization and expecting over the other suppliers' messages*, must be at least as great from telling the truth as from lying. A problem with the Bayesian Nash equilibrium in a situation with multiple suppliers is that it can result in contracts for which, in addition to the truth-telling equilibrium, there may exist a non-truth-telling equilibrium which all of the suppliers prefer but which makes the buyer strictly worse off (see (Demski and Sappington, 1984)). In that case, the suppliers can implicitly collude to implement their preferred non-truth-telling equilibrium, rather than the buyer's preferred truth-telling equilibrium. To avoid that situation, we use the Dominant Strategy equilibrium concept which is usually more restrictive. It states that each supplier's expected profit, *given his cost realization*, must be at least as great from telling the truth as from lying, *regardless of the other suppliers' messages*. With the Dominant Strategy equilibrium, there is no possibility of implicit collusion, because each supplier prefers truth-telling regardless of what the other suppliers decide to do (see (Kreps, 1990)). Fortunately, given our model, the use of the more restrictive Dominant Strategy equilibrium is costless because the optimal solution using the Bayesian Nash equilibrium can be equivalently implemented in dominant strategies. For sufficient conditions for this to be true, see (Mookherjee, 1992).

$$\max_{\left\{x_{2kj}^{A},x_{1kj}^{A},x_{2kj}^{B},x_{1kj}^{B}\right\}} \sum_{j=0}^{n}\sum_{k=0}^{n}\begin{pmatrix} R\left\{\min\left(\left(n-k\right)x_{2kj}^{A}+kx_{1kj}^{A};\left(n-j\right)x_{2kj}^{B}+jx_{1kj}^{B}\right)\right\} \\ -kT_{1kj}^{A}-\left(n-k\right)T_{2kj}^{A}-jT_{1kj}^{B}-\left(n-j\right)T_{2kj}^{B}\end{pmatrix}\times P\left(k,j,n\right)-nFC\left(n\right).$$

subject to, for each feasible $\left(k,j\right)$ cost realization:

$$\begin{aligned}
&T_{1kj}^{A}-c_1\left(x_{1kj}^{A}\right)^2-FC\left(n\right)\geq 0\\
&T_{2kj}^{A}-c_2\left(x_{2kj}^{A}\right)^2-FC\left(n\right)\geq 0\\
&T_{1kj}^{B}-c_1\left(x_{1kj}^{B}\right)^2-FC\left(n\right)\geq 0\\
&T_{2kj}^{B}-c_2\left(x_{2kj}^{B}\right)^2-FC\left(n\right)\geq 0
\end{aligned}\qquad \text{(LL)}$$

$$\begin{aligned}
&T_{1kj}^{A}-c_1\left(x_{1kj}^{A}\right)^2-FC\left(n\right)\geq T_{2(k-1)j}^{A}-c_1\left(x_{2(k-1)j}^{A}\right)^2-FC\left(n\right)\\
&T_{2(k-1)j}^{A}-c_2\left(x_{2(k-1)j}^{A}\right)^2-FC\left(n\right)\geq T_{1kj}^{A}-c_2\left(x_{1kj}^{A}\right)^2-FC\left(n\right)\\
&T_{1kj}^{B}-c_1\left(x_{1kj}^{B}\right)^2-FC\left(n\right)\geq T_{2k(j-1)}^{B}-c_1\left(x_{2k(j-1)}^{B}\right)^2-FC\left(n\right)\\
&T_{2k(j-1)}^{B}-c_2\left(x_{2k(j-1)}^{B}\right)^2-FC\left(n\right)\geq T_{1kj}^{B}-c_2\left(x_{1kj}^{B}\right)^2-FC\left(n\right)
\end{aligned}\qquad \text{(TT)}$$

where now:

$x_{2kj}^{A}$ = the number of units a part A supplier with a *reported* cost realization of $c_2$ is directed to provide given that there were a total of *k* part A suppliers *who reported* $c_1$ realizations, *n-k* part A suppliers *who reported* $c_2$ realizations, *j* part B suppliers *who reported* $c_1$ realizations, and *n-j* part B suppliers *who reported* $c_2$ realizations.

$T_{2kj}^{A}$ = the compensation to a part A supplier for reporting $c_2$ and producing $x_{2kj}^{A}$.

The order sizes and compensations for the part A suppliers with $c_1$ cost realizations and for B suppliers are defined analogously.

The Limited Liability constraints (LL) indicate that unless the production and compensation plans offered to each supplier assure at least zero profit for each cost realization, he will leave the supply chain. The Truth-Telling constraints (TT) restrict (without loss of generality) our attention to truth-telling equilibria. For example, consider the viewpoint of an A supplier with a low-cost realization. Assume that that the other *n-1* part A suppliers and the *n* part B suppliers report that there are *k-1* low-cost part A suppliers, *n-k* high-cost part A suppliers, *j* low-cost part B suppliers and *n-j* high-cost part B suppliers. Then the TT constraint ensures that low-cost part A supplier should prefer truthfully revealing that he is low-cost and having the buyer believe that she is facing the *(k,j)* cost realization rather than lying and having the buyer believe that she is facing the *(k-1,j)* cost realization.

Notice that a part A supplier's reporting decision affects both what he reports his cost realization to be and the total number of reported part A low-cost realizations. Further, the messages reported by the other suppliers affect each supplier's production and compensation plan, but not that particular supplier's cost of fulfilling the plan. That is, the other suppliers' cost realizations only have an indirect effect on each supplier's utility (i.e., through the offered plan).

The following Lemma uses the Individual Rationality and Truth-Telling constraints to provide an initial characterization of the optimal solution to the Second-Best problem formulation:

<u>Lemma 2:</u> *The optimal solution to the Second-Best problem has the following characteristics:*

1. $$T_{1kj}^{A} - c_1\left(x_{1kj}^{A}\right)^2 - FC(n) > 0 \ \forall k,j \text{ where } k \neq 0$$ $$T_{1kj}^{B} - c_1\left(x_{1kj}^{B}\right)^2 - FC(n) > 0 \quad \forall k,j \text{ where } j \neq 0$$

2. $$T_{2(k-1)j}^{A} - c_2\left(x_{2(k-1)j}^{A}\right)^2 - FC(n) = 0 \ \forall j,k \text{ where } k \neq 0$$ $$T_{2k(j-1)}^{B} - c_2\left(x_{2k(j-1)}^{B}\right)^2 - FC(n) = 0 \quad \forall k,j \text{ where } j \neq 0$$

3. $x_{1kj}^{A} \geq x_{2(k-1)j}^{A} \ \forall j,$ and $k \neq 0$ and $x_{1kj}^{B} \geq x_{2k(j-1)}^{B} \quad \forall k,$ and $j \neq 0$ [22]

4. $$T_{1kj}^{A} = FC(n) + c_1\left(x_{1kj}^{A}\right)^2 + c_2\left(x_{2(k-1)j}^{A}\right)^2 - c_1\left(x_{2(k-1)j}^{A}\right)^2 \quad \forall j,k \text{ where } k \neq 0$$ $$T_{1kj}^{B} = FC(n) + c_1\left(x_{1kj}^{B}\right)^2 + c_2\left(x_{2k(j-1)}^{B}\right)^2 - c_1\left(x_{2k(j-1)}^{B}\right)^2 \quad \forall k,j \text{ where } j \neq 0.$$

[22] Our subsequent analysis ignores the constraint embodied in Result 3 of Lemma 2. However, the resulting optimal solution satisfies it as a strict inequality, so ignoring the constraint is without loss of generality.

The characterization in Lemma 2 is typical of adverse selection problems such as ours, in which the agents (i.e., suppliers) have superior information to that possessed by the principal (i.e., buyer). The low-cost suppliers' production is at least as great as the high-cost suppliers' (Result 3), the high-cost suppliers just earn their outside reservation wage (Result 2), and the low-cost suppliers earn more than their outside reservation wage (Result 1). This extra amount earned by the low-cost supplier is referred to as an informational rent.[23] This rent is the cost to the buyer associated with the supplier-incentive problem. In order to properly coordinate production, the buyer must elicit the true cost realizations from the A and B suppliers. But to do so, the buyer must share some of her profit with the low-cost suppliers (i.e., bribe the supplier with the informational rent). From Result 4 of Lemma 2, the buyer can reduce the low-cost A supplier's informational rent by decreasing the *high-cost* A supplier's production $\left(x_{2(k-1)j}^{A}\right)$– but this will distort the high-cost suppliers' production away from First-Best and result in unbalanced A and B parts production. Of course, the resulting production mismatch can be mitigated by adjusting the low-cost A supplier's production and/or both B suppliers' production. But inevitably, the presence of the supplier-incentive problem forces the buyer to trade-off informational rents and production efficiency. Proposition 1, below, further characterizes the Second-Best Problem.

*Proposition 1: At optimality,* $(n-k)x_{2kj}^{A}+kx_{1kj}^{A}=(n-j)x_{2kj}^{B}+jx_{1kj}^{B}=\overline{X}^{I}\quad\forall j,k$ *and the Second-Best problem can be restated as*

$$\max_{\left\{\overline{X}^{I}\right\}}\left[\max_{\left\{x_{1kj}^{AI},x_{1kj}^{BI}\right\}}\sum_{j=0}^{n}\sum_{k=0}^{n}\left(\begin{array}{l}R\overline{X}^{I}-kc_{1}\left(x_{1kj}^{AI}\right)^{2}-(n-k)\left(\dfrac{\overline{X}^{I}-kx_{1kj}^{AI}}{(n-k)}\right)^{2}(c_{2}+\Delta)\\ -jc_{1}\left(x_{1kj}^{BI}\right)^{2}-(n-j)\left(\dfrac{\overline{X}^{I}-jx_{1kj}^{BI}}{(n-j)}\right)^{2}(c_{2}+\Delta)\end{array}\right)\times P(k,j,n)-nFC(n)\right]\qquad(3)$$

*where* $\Delta=p(c_{2}-c_{1})/(1-p)$ *and the I superscript refers to the optimal Second-Best (or Incentive) solution.*

[23] For the low-cost A supplier the informational rent is $(c_{2}-c_{1})\left(x_{2(k-1)j}^{A}\right)^{2}$, see Result 4.

Proposition 1 demonstrates that the Second-Best problem is exactly the same as the First-Best, except that, in the Second-Best case, $c_2$ is everywhere replaced by $c_2+\Delta$. Hence, the same is true for the Second-Best solution. $c_2+\Delta$ is referred to as the buyer's virtual cost (see (Laffont and Martimort, 2002)). The $\Delta$ represents the additional cost which the buyer must bear as a result of the supplier-incentive problems. Thus, the optimal solution for the Second-Best problem is:[24]

$$
\begin{aligned}
\overline{X}^I_{kj} &= \frac{Rn\left(c_1(1-p)+(c_2-c_1)k/n\right)\left(c_1(1-p)+(c_2-c_1)j/n\right)}{2c_1(c_2-pc_1)\left(2c_1(1-p)+(j/n+k/n)(c_2-c_1)\right)}\\
x^{AI}_{1kj} &= \frac{R\left(c_1(1-p)+(c_2-c_1)j/n\right)}{2c_1\left(2c_1(1-p)+(j/n+k/n)(c_2-c_1)\right)}\\
x^{AI}_{2kj} &= \frac{R(1-p)\left(c_1(1-p)+(c_2-c_1)j/n\right)}{2(c_2-pc_1)\left(2c_1(1-p)+(j/n+k/n)(c_2-c_1)\right)}\\
x^{BI}_{1kj} &= \frac{R\left(c_1(1-p)+(c_2-c_1)k/n\right)}{2c_1\left(2c_1(1-p)+(j/n+k/n)(c_2-c_1)\right)}\\
x^{BI}_{2kj} &= \frac{R(1-p)\left(c_1(1-p)+(c_2-c_1)k/n\right)}{2(c_2-pc_1)\left(2c_1(1-p)+(j/n+k/n)(c_2-c_1)\right)}
\end{aligned}
\tag{4}
$$

The difference between the optimal solution to the First-Best problem (1) and the Second-Best problem (4) is due solely to the supplier-incentive problem. Analogous to the First-Best case, the buyer's profit for the cost realization $(k,j)$ is:

$$\pi^I_{kj} = R\overline{X}^I - \left(\overline{X}^I\right)^2 c_1(c_2+\Delta)\frac{2nc_1+(k+j)(c_2+\Delta-c_1)}{\left(nc_1+k(c_2+\Delta-c_1)\right)\left(nc_1+j(c_2+\Delta-c_1)\right)} - nFC(n).$$

After substituting in the optimal $\overline{X}^F_{kj}$, we obtain

$$\pi^I_{kj} = \frac{R^2\left(nc_1+k(c_2+\Delta-c_1)\right)\left(nc_1+j(c_2+\Delta-c_1)\right)}{4c_1(c_2+\Delta)\left(2nc_1+(j+k)(c_2+\Delta-c_1)\right)} - nFC(n) = \frac{R\overline{X}^I_{kj}}{2} - nFC(n).$$

The buyer's total expected profit is:

---

[24] Recall that we initially ignored Result 3 from Lemma 2 in our subsequent analysis. However, it is straightforward to verify that the optimal solution in (4) satisfies it as a strict inequality for all *k, j*.

$$\frac{R}{2}\sum_{j=0}^{n}\sum_{k=0}^{n}\bar{X}_{kj}^{I}P(k,j,n)-nFC(n). \qquad (5)$$

Thus, for each $(k,j)$ realization the incentive efficiency effect of the supplier-incentive problem is captured by the distortion in total production - $\bar{X}_{kj}^{I}$ vs. $\bar{X}_{kj}^{F}$. Therefore, we next analyze the production distortions, beginning with the distortions in the individual supplier's orders.

**b. Production distortions**

For the high-cost part A supplier (the same is true for the part B supplier) it is straight-forward to show that:

$$\begin{aligned}
&x_{2kj}^{AF}/x_{2kj}^{AI}>1\\
&\frac{\partial}{\partial n}\left(x_{2kj}^{AF}/x_{2kj}^{AI}\right)<0\\
&\lim_{n\to\infty}\left(x_{2kj}^{AF}/x_{2kj}^{AI}\right)>1
\end{aligned} \qquad (6)$$

The supplier-incentive problem causes each high-cost supplier to always produce less than First-Best, regardless of the cost realization. For any given *(k,j)* cost realization, this relative distortion in individual production is mitigated by an increase in the number of suppliers, but is never eliminated. Thus, increasing the number of suppliers mitigates the distortion caused by the supplier-incentive problem for the high-cost supplier and, hence, has a positive incentive effect on individual high-cost supplier production.

For the low-cost A (and B) suppliers one can also show that:

$$\begin{aligned}
&\text{when } k>j,\ x_{1kj}^{AF}/x_{1kj}^{AI}>1,\ x_{1kj}^{BF}/x_{1kj}^{BI}<1;\\
&\text{when } k=j,\ x_{1kj}^{AF}/x_{1kj}^{AI}=x_{1kj}^{BF}/x_{1kj}^{BI}=1;\\
&\text{when } k<j,\ x_{1kj}^{AF}/x_{1kj}^{AI}<1,\ x_{1kj}^{BF}/x_{1kj}^{BI}>1;\\
&\lim_{n\to\infty}\left(x_{1kj}^{AF}/x_{1kj}^{AI}\right)=\lim_{n\to\infty}\left(x_{1kj}^{BF}/x_{1kj}^{BI}\right)=1.
\end{aligned} \qquad (7)$$

The Second-Best solution is such that the low-cost part A (B) supplier will produce efficiently only when $k = j$, when the proportion of low-cost part A and B suppliers is the same. However, given the supplier-incentive problem, the low-cost A (B) supplier will produce more (less) than the efficient amount when $k < j$ and less (more) than the efficient amount when $k > j$. As with the high-cost supplier, the relative production distortion for the low-cost supplier varies in *n*. However there are two differences between the high and low-cost suppliers. First, for any fixed *(k,j)* realization, this distortion disappears as *n* gets large for the low-cost supplier but not for the high-cost supplier. Second, the effect on the low-cost relative production distortion depends on the specific parameters, while it is unambiguous for the high-cost supplier.

To put (6) and (7) in context, recall that that our two main assumptions are: a supplier-incentive or adverse selection problem (i.e., privately-informed and self-interested suppliers) and a production-balancing problem (i.e., the buyer pays for but obtains no value from any A parts delivered in excess of B parts (or vice versa)). With no supplier-incentive problem, First-Best can always be achieved and there is no distortion in production. With a supplier-incentive problem but no production-balancing problem, which is the standard adverse selection case examined in the literature, the low-cost supplier always produces the First-Best quantity, while the high-cost supplier always produces less than First-Best, and the individual supplier production distortions are *invariant* to the number of suppliers.[25] Hence, in this case the number of suppliers has no incentive-related effect on individual relative production distortion. With *both* the supplier-incentive problem and the production-balancing problem, both low-cost and high-cost suppliers produce inefficiently and each individual supplier's relative production distortion varies in the number of suppliers. Thus, it is the existence of the supplier-incentive problem that gives rise to production distortions, but it is the presence of the production-balancing problem, normally associated with assembly operations, which results in individual supplier relative production distortions that vary in the number of suppliers.

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[25] A model comparable to ours, but without the production-balancing problem is: there is an A end-product, and a B end-product; each A end-product is made up of one A part; each B end-product is made up of one B part; each end-product is sold for \$*R/2* per unit. In this case, First-Best low and high-cost productions are $(R/4c_1, R/4c_2)$ and Second-Best low and high-cost production are $(R/4c_1, R/(4c_2+\Delta))$. Relative production distortion is thus independent of *n*. Like us, (Levin, 2003) finds inefficient production for all cost realizations in a model of adverse selection but with incomplete contracting and repeated play.

The reason our production distortion results are different from the usual is that, as with all supplier-incentive problems, the buyer trades off the cost of inefficient production against informational rent by distorting the high-cost production schedule. In the absence of the production-balancing problem, the buyer can distort the high-cost suppliers' production without incurring any production-balancing cost (e.g., paying for excess parts without being able to use them in production) and thus without the need to distort the low-cost suppliers' production to mitigate that balancing cost.[26] However, with the production-balancing problem, distorting the high-cost suppliers' production incurs the additional cost of unbalanced production. To avoid this, the buyer finds it optimal for most $(k, j)$ cost realizations to distort the low-cost suppliers' production as well.

In summary, we have found that the high-cost supplier always produces less than First-Best while the low-cost supplier's production distortion depends upon the cost realization. Further, while increasing *n* mitigates the high-cost relative production distortion caused by the supplier-incentive problem; it can either mitigate or exacerbate, depending on the problem parameters, the same effect on the low-cost relative production distortion. Whether the overall effect of increasing the number of suppliers is to enhance or reduce the overall relative production distortion depends on its aggregate effect on $\bar{X}^I_{kj}$ relative to $\bar{X}^F_{kj}$. In order to address this issue we must first determine the relative sizes of $\bar{X}^I_{kj}$ and $\bar{X}^F_{kj}$.

Proposition 2 below shows that the underproduction by high-cost suppliers more than compensates for any overproduction by low-cost suppliers and hence, that Second-Best total production for *any* given $(k,\ j) \neq (n,n)$ realization is always *less* than First-Best.

<u>*Proposition 2*</u>

1. $\bar{X}^I_{kj} / \bar{X}^F_{kj} < 1 \quad \forall (k, j) \neq (n,n)$ *and* $\bar{X}^F_{nn} / \bar{X}^I_{nn} = 1$

[26] This is why, in the absence of the production-balancing problem, the cost of eliciting the truth from any supplier is independent of the number of other suppliers.

2. $\lim_{n\to\infty}\left(\overline{X}_{kj}^{I} / \overline{X}_{kj}^{F}\right) < 1 \quad \forall k, j$

Thus, the supplier-incentive problem results in total production for each $(k,\ j) \neq (n,n)$ realization which is less than First-Best. In particular, the production distortion for any fixed $(k,\ j)$ cost realization does not disappear as the number of suppliers gets large. Proposition 3 below establishes how this total production distortion is affected by the number of suppliers.

*Proposition 3*[27]

1. $\dfrac{\partial}{\partial n}\dfrac{\overline{X}_{kj}^{I}}{\overline{X}_{kj}^{F}} \geq 0 \quad \forall (k, j)$

2. $\dfrac{\partial \overline{X}_{kj}^{I}}{\partial n} \Bigg/ \dfrac{\overline{X}_{kj}^{I}}{n} \geq \dfrac{\partial \overline{X}_{kj}^{F}}{\partial n} \Bigg/ \dfrac{\overline{X}_{kj}^{F}}{n} \quad \forall (k, j).$

From Proposition 2 we know that the Second-Best total production for each *(k,j)* cost realization is less than or equal to the First-Best, resulting in a production distortion and a loss of efficiency. Result 1 in Proposition 3 shows that an increase in the number of suppliers *reduces* the relative distortion in total production for each $(k,\ j)$ realization. It should be noted that increasing the number of suppliers results in a direct production effect, the increase in total production for both Second and First-Best for each $(k,\ j)$ realization. However, Second-Best total production increases relatively faster than First-Best production and, thus, the relative distortion for each $(k,\ j)$ realization caused by the supplier-incentive problem is *decreasing* in $n$. Therefore, for a buyer facing privately-informed and self-interested suppliers, the choice of *n* has an additional incentive effect beyond the direct production effect. For such a buyer choosing *n* is a way of mitigating the cost of dealing with privately-informed and self-interested suppliers.

[27] For this part of the analysis we allow *n* to take non-integer values. While clearly not the case, this assumption still captures the essence of the problem.

Note that, from Part 2 of Proposition 2, for any fixed $(k,\ j)$ realization, while increasing *n* reduces this production distortion, it does not eliminate it.

Result 2 of Proposition 3 represents the distortion somewhat differently. In Result 2 we are concerned with the change in total production caused by a change in *n* relative to the average total production per supplier, that is, the elasticity of production for each $(k,\ j)$ realization as a function of the number of suppliers. Again, the effect of an increase in *n* is greater on Second-Best total production than on First-Best total production. Proposition 3 thus shows that, in the presence of a supplier-incentive problem, there is a positive incentive efficiency effect associated with increasing the number of suppliers *–for each* $(k,\ j)$ *realization it reduces the relative distortion in total production.*

The basic intuition for Proposition 3 can be explained as follows. Because of the production-balancing problem, each supplier's optimal production will depend upon the other suppliers' cost realizations. As the number of suppliers of each part increases, what any one supplier reports about his cost realization becomes relatively less important to the buyer in determining the optimal production schedules for the other suppliers. This, in turn, reduces his informational rent and the relative distortion in total production required to induce him to honestly reveal his cost realization (Proposition 3). As noted earlier, without the production-balancing problem, each supplier's production is independent of the number of the other suppliers and their cost realizations, and thus, increasing the number of suppliers has no effect on the suppliers' contracts or production. Hence, supply chains which are subject to the production-balancing problem will exhibit this efficiency enhancing incentive-related effect associated with the number of suppliers.

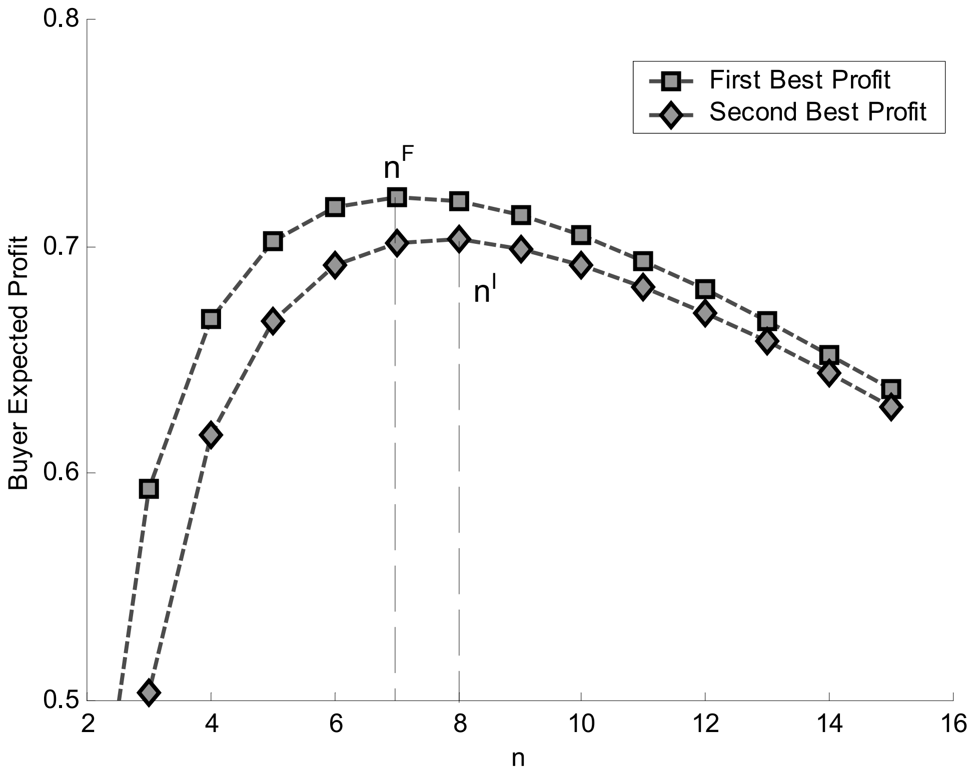
### c. Implications for the optimal number of suppliers

In summary, Propositions 2 and 3 establish that, for each $(k,\ j)$ cost realization, total Second-Best production is *less than or equal to* First-Best but that increasing the number of suppliers reduces this relative production distortion. This would seem to suggest that, given the production-balancing problem, the buyer facing a supplier-incentive problem *has a greater demand for suppliers* than does the same buyer not facing a supplier-incentive problem. That is, these results suggest that the optimal number of suppliers under Second-Best should be at least as great as under First-Best. The difficulty in establishing this analytically is that varying *n* has three effects:

1. it changes the optimal First and Second-Best total productions for every $(k,\ j)$ realization,
2. it "fills in" the distribution by increasing the set of $(k,\ j)$ realizations,
3. it changes the probabilities associated with every $(k, j)$ outcome.

Proposition 3 addressed the first effect. We are unable to characterize analytically the second and third effects, and thus cannot analytically establish this conjecture. Thus, we next turn to numerical analysis. To best test this conjecture, consider the First and Second-Best problems analyzed so far, but with two modifications. First, we assume that the buyer has a fixed amount of output that she needs to produce regardless of the suppliers' cost realizations, $\bar{X}$. This is a special case of the endogenous production scenario which we have studied so far. The reason for this modification is that the optimal First and Second-Best productions will, in general, be different. Different production will imply different optimal numbers of suppliers. In order to isolate the demand for suppliers strictly arising from the supplier-incentive problem, we keep the production for First and Second-Best the same, $\bar{X}$. Second, we assume that *FC(n)=F,* so that there is a fixed cost to production for each supplier, or alternatively, the buyer incurs a communication/coordination cost which is linear in the number of suppliers. Extensive numerical analysis produces consistent results, the optimal number of suppliers for Second-Best is always greater than or equal to that for First-Best. Figure below illustrates this for the following parameter values: $\left(c_1 = .3, c_2 = 1, p = .5, R = 1, F = .02, \bar{X} = 1\right)$.

The figure indicates that for the given parameter values the optimal number of suppliers under First-Best is 7, while under Second-Best it is 8. This further demonstrates that the number of suppliers has an incentive-efficiency effect. Given a production-balancing problem, the presence of privately-informed and self-interested suppliers make it optimal for the buyer to have a larger supplier base than he would have if the suppliers were not privately-informed and self-interested.




## 5. Conclusion

In this paper we study the effect that multi-sourcing has on an assembler who needs to coordinate her privately-informed and self-interested suppliers. Much of the literature has been devoted to studying the effect that the number of supplier's has on: (dis)economies of scale of production, risk of disruption, and cost of communication. In this paper we point out that there is an additional effect that must be considered when the buyer chooses her number of suppliers. The number of suppliers affects the relative size of production distortions caused by the supplier-incentive problem. Increasing the number of suppliers mitigates this production distortion. The reason is that as the number of suppliers of each part increases, what any one supplier reports has a relatively smaller effect in determining the production schedules. This, in turn, reduces the amount of informational rent which any supplier can extract from the buyer and hence reduces the amount by which the buyer has to distort production in order to reduce the informational rent.

Simply stated, our finding is that by increasing the number of suppliers the buyer is decreasing their importance, thereby decreasing their informational rents and improving efficiency. In a very different model, one with incomplete contracting and buyer hold-up behavior, (Bakos and Brynjolfsson, 1993) found that *decreasing* the number of suppliers *increased* their relative bargaining power, thereby *increasing* their incentive to invest in relationship-specific capital, thereby *improving* efficiency. Our work, together with (Bakos and Brynjolfsson, 1993), indicates that the number of suppliers in a supply chain can have subtle efficiency effects and that the appropriate number of suppliers depends on the underlying incentive problems. Moreover, our work demonstrates the importance of considering functional externalities among product parts that often exist but are typically ignored in much of the procurement literature.

Our line of inquiry is a first step towards a deeper understanding of incentive effects in multi-sourcing arrangements. While we considered only one (but perhaps the most natural) externality that can exist among a buyer and her suppliers, other types of externalities can be studied. In our work the assembly system had only two tiers while in practice there are often multi-tiered assembly systems (see, for example, (Bernstein and DeCroix, Forthcoming)). Hence, one interesting question is: what incentive-related effect does the number of suppliers in each tier have on overall efficiency? Analysis of incentive effects under relational (multi-period and incomplete) contracts could also prove a fruitful direction.

## Appendix

Proof of Lemma 2: We only prove these results for the part A suppliers since the proof for the part B suppliers is analogous.

Result 1. Note that

$T_{1kj}^{A} - c_1\left(x_{1kj}^{A}\right)^2 - FC \geq T_{2(k-1)j}^{A} - c_1\left(x_{2(k-1)j}^{A}\right)^2 - FC(n) > T_{2(k-1)j}^{A} - c_2\left(x_{2(k-1)j}^{A}\right)^2 - FC(n) \geq 0$. The first inequality follows from A's (TT) Constraint, the second from $c_1 < c_2$ and the third from A's (LL) Constraint.

Result 2. For high-cost A suppliers assume otherwise (i.e., the high-cost (LL) Constraint is not binding) and suppose that we start with an optimal solution so that all of the constraints are satisfied. Reduce both $T_{1kj}^{A}$ and $T_{2(k-1)j}^{A}$ by $\varepsilon$. None of the constraints will be violated by the change and the manufacturer increases his objective function. This is a contradiction.

Result 3. From the (TT) constraints for the part A suppliers,

$c_1\left(x_{2(k-1)j}^{A}\right)^2 - c_1\left(x_{1kj}^{A}\right)^2 \geq T_{2(k-1)j}^{A} - T_{1kj}^{A} \geq c_2\left(x_{2(k-1)j}^{A}\right)^2 - c_2\left(x_{1kj}^{A}\right)^2$. The result then follows from the fact that $c_1 < c_2$.

Result 4. The (TT) constraints for the part A suppliers and Result 2 above imply that

$T_{1kj}^{A} \geq FC(n) + c_1\left(x_{1kj}^{A}\right)^2 + c_2\left(x_{2(k-1)j}^{A}\right)^2 - c_1\left(x_{2(k-1)j}^{A}\right)^2$. The manufacturer is interested in minimizing $T_{1kj}^{A}$, hence, $T_{1kj}^{A} = FC(n) + c_1\left(x_{1kj}^{A}\right)^2 + c_2\left(x_{2(k-1)j}^{A}\right)^2 - c_1\left(x_{2(k-1)j}^{A}\right)^2$ and the result follows.

Proof of Proposition 1: We first need to establish that in the Second-Best problem,

$(n-k)x_{2kj}^{A} + kx_{1kj}^{A} = (n-j)x_{2kj}^{B} + jx_{1kj}^{B} = \bar{X}^{I} \quad \forall j,k$.

Proof: Assume the contrary so that for some $(k', j')$ realization

$(n-k')x_{2k'j'}^{A} + k'x_{1k'j'}^{A} > (n-j')x_{2k'j'}^{B} + j'x_{1k'j'}^{B}$. In this case, only $(n-j')x_{2k'j'}^{B} + j'x_{1k'j'}^{B}$ end-products will be produced and $(n-k')x^A_{2k'j'}+k'x^A_{1k'j'}-(n-j')x^B_{2k'j'}-j'x^B_{1k'j'}$ units of part A will be thrown away. Using the results of Lemma 2, the Second-Best problem can then be restated as:

$$\max_{\left\{x^A_{2kj},x^A_{1kj},x^B_{2kj},x^B_{1kj}\right\}}\left\{\begin{array}{l}\displaystyle\sum_{j\neq j'}^{n}\sum_{k=0}^{n}\left(\begin{array}{l}R\min\left((n-k)x^A_{2kj}+kx^A_{1kj};(n-j)x^B_{2kj}+jx^B_{1kj}\right)\\-k\left(c_1\left(x^A_{1kj}\right)^2+c_2\left(x^A_{2(k-1)j}\right)^2-c_1\left(x^A_{2(k-1)j}\right)^2\right)\\-(n-k)c_2\left(x^A_{2kj}\right)^2-j\left(c_1\left(x^B_{1kj}\right)^2+c_2\left(x^B_{2k(j-1)}\right)^2-c_1\left(x^B_{2k(j-1)}\right)^2\right)\\-(n-j)c_2\left(x^B_{2kj}\right)^2\end{array}\right)\times P(k,j,n)\\
+\displaystyle\sum_{j=0}^{n}\sum_{k\neq k'}^{n}\left(\begin{array}{l}R\min\left((n-k)x^A_{2kj'}+kx^A_{1kj'};(n-j')x^B_{2kj'}+j'x^B_{1kj'}\right)\\-k\left(c_1\left(x^A_{1kj'}\right)^2+c_2\left(x^A_{2(k-1)j'}\right)^2-c_1\left(x^A_{2(k-1)j'}\right)^2\right)\\-(n-k)c_2\left(x^A_{2kj'}\right)^2-j\left(c_1\left(x^B_{1kj'}\right)^2+c_2\left(x^B_{2k(j'-1)}\right)^2-c_1\left(x^B_{2k(j'-1)}\right)^2\right)\\-(n-j')c_2\left(x^B_{2kj'}\right)^2\end{array}\right)\\
\qquad\times P(k,j,n)\\
+\left(\begin{array}{l}R\left((n-j')x^B_{2k'j'}+j'x^B_{1k'j'}\right)\\-k'\left(c_1\left(x^A_{1k'j'}\right)^2+c_2\left(x^A_{2(k'-1)j'}\right)^2-c_1\left(x^A_{2(k'-1)j'}\right)^2\right)\\-(n-k')c_2\left(x^A_{2k'j'}\right)^2-j\left(c_1\left(x^B_{1k'j'}\right)^2+c_2\left(x^B_{2k'(j'-1)}\right)^2-c_1\left(x^B_{2k'(j'-1)}\right)^2\right)\\-(n-j')c_2\left(x^B_{2k'j'}\right)^2\end{array}\right)\times P(k,j,n)-nFC(n)\end{array}\right\}$$

Notice that $x^A_{2k'j'}$ and $x^A_{1k'j'}$ appear in the objective function only with negative coefficients. Hence, their optimal values will be zero. On the other hand, $x^B_{2k'j'}$ and $x^B_{1k'j'}$ appear with both positive and negative coefficients. Hence their optimal values will be non-negative. But this contradicts the assumption that $(n-k')x^A_{2k'j'}+k'x^A_{1k'j'}>(n-j')x^B_{2k'j'}+j'x^B_{1k'j'}$.

Given this result, the Second-Best problem can be restated as:

$$\max_{\left\{\overline{X}^I_{kj}\right\}}\left[\begin{array}{l}\displaystyle\max_{\left\{x^A_{2kj},x^A_{1kj},x^B_{2kj},x^B_{1kj}\right\}}\sum_{j=0}^{n}\sum_{k=0}^{n}\left(\begin{array}{l}R\overline{X}^I_{kj}-kc_1\left(x^{AI}_{1kj}\right)^2-k(c_2-c_1)\left(x^{AI}_{2(k-1)j}\right)^2-(n-k)c_2\left(x^{AI}_{2kj}\right)^2\\-jc_1\left(x^{BI}_{1kj}\right)^2-j(c_2-c_1)\left(x^{BI}_{2k(j-1)}\right)^2-(n-j)c_2\left(x^{BI}_{2kj}\right)^2\end{array}\right)\\
\qquad\times P(k,j,n)-nFC(n)\end{array}\right] \tag{8}$$

For a given $(k, j)$ cost realization, the buyer's Second-Best objective function is:

$$\pi_{kj}^{I} = R\overline{X}_{kj}^{I} - kc_1\left(x_{1kj}^{AI}\right)^2 - k\left(c_2 - c_1\right)\left(x_{2(k-1)j}^{AI}\right)^2 - \left(n-k\right)c_2\left(x_{2kj}^{AI}\right)^2$$
$$- jc_1\left(x_{1kj}^{BI}\right)^2 - j\left(c_2 - c_1\right)\left(x_{2k(j-1)}^{BI}\right)^2 - \left(n-j\right)c_2\left(x_{2kj}^{BI}\right)^2$$

Given that the Second-Best objective function sums over *j* and *k*, we can re-express each $\pi_{kj}^{I}$ term in (8) by moving out the terms $k(c_2 - c_1)x_{2(k-1)j}^{AI}$ and $j(c_2 - c_1)x_{2k(j-1)}^{BI}$ to $\pi_{(k-1)j}^{I}$ and $\pi_{k(j-1)}^{I}$, and moving in the terms $(k+1)(c_2 - c_1)x_{2kj}^{AI}$ and $(j+1)(c_2 - c_1)x_{2kj}^{BI}$ from $\pi_{(k+1)j}^{I}$ and $\pi_{k(j+1)}^{I}$. In doing so we have to adjust for the differences in probabilities associated with these terms. The probability mass $P(k+1, j, n)$ associated with the term $(k+1)(c_2 - c_1)x_{2kj}^{AI}$ can be written as[28]

$$P\left(k+1, j, n\right) = \frac{n-k}{k+1}\frac{p}{1-p}\frac{n!}{k!\left(n-k\right)!}p^k\left(1-p\right)^{n-k}\frac{n!}{j!\left(n-j\right)!}p^j\left(1-p\right)^{n-j} = \frac{n-k}{k+1}\frac{p}{1-p}\Pr(k, j, n)$$

Hence, we obtain

$$\begin{aligned}\pi_{kj}^{I} &= R\overline{X}_{kj}^{I} - kc_1\left(x_{1kj}^{AI}\right)^2 - \left(n-k\right)\left(c_2 - c_1\right)\frac{p}{1-p}\left(x_{2kj}^{AI}\right)^2 - \left(n-k\right)c_2\left(x_{2kj}^{AI}\right)^2 \\ &\quad - jc_1\left(x_{1kj}^{BI}\right)^2 - \left(n-j\right)\left(c_2 - c_1\right)\frac{p}{1-p}\left(x_{2kj}^{BI}\right)^2 - \left(n-j\right)c_2\left(x_{2kj}^{BI}\right)^2 \\ &= R\overline{X}_{kj}^{I} - kc_1\left(x_{1kj}^{AI}\right)^2 - \left(n-k\right)\left(x_{2kj}^{AI}\right)^2\left(c_2 + \Delta\right) - jc_1\left(x_{1kj}^{BI}\right)^2 - \left(n-j\right)\left(x_{2kj}^{BI}\right)^2\left(c_2 + \Delta\right).\end{aligned} \tag{9}$$

where $\Delta = p(c_2 - c_1)/\left(1-p\right)$. The Proposition follows immediately.

Proof of Proposition 2:

Result 1: Recall that the Second-Best objective function is the same as the First-Best, except with $c_2 + \Delta, \Delta > 0$ replacing $c_2$. Hence $\overline{X}_{kj}^{FB} = \overline{X}_{kj}^{I}$ at $\Delta = 0$. Therefore, if $\overline{X}_{kj}^{I}$ is everywhere decreasing in $\Delta$, then $\overline{X}_{kj}^{FB} > \overline{X}_{kj}^{I}$. It is straightforward to show that

$$\frac{\partial \overline{X}_{kj}^{I}}{\partial \Delta} = -\frac{\dfrac{(n-j)(n-k)}{\left(c_2 + \Delta\right)^2} + \dfrac{(j-k)^2 n^2}{((2n-k-j)c_1 + (j+k)\left(c_2 + \Delta\right))^2}}{2(2n-k-j)} < 0 \quad \forall k, j \text{ s.t. } j \neq k \neq n$$

[28] The term $(j+1)(c_2 - c_1)x_{2kj}^{B}$ is treated analogously.

Hence $\overline{X}_{kj}^{F} > \overline{X}_{kj}^{I} \quad \forall (j,k) \neq (n,n)$.

Result 2 follows immediately from (1) and (4).

Proof of Proposition 3: Both Results 1 and 2 can be established together. Assume that there are $n_A$ A suppliers and $n_B$ B suppliers. It is then straightforward to show that:

$$\overline{X}_{kj}^{F} = \frac{R\left(n_A c_1 + k\left(c_2 - c_1\right)\right)\left(n_B c_1 + j\left(c_2 - c_1\right)\right)}{2c_1 c_2\left(\left(n_A + n_B\right)c_1 + \left(j+k\right)\left(c_2 - c_1\right)\right)}.$$

$$\overline{X}_{kj}^{I} = \frac{R\left(n_A c_1 + k\left(c_2 + \Delta - c_1\right)\right)\left(n_B c_1 + j\left(c_2 + \Delta - c_1\right)\right)}{2c_1\left(c_2 + \Delta\right)\left(\left(n_A + n_B\right)c_1 + \left(j+k\right)\left(c_2 + \Delta - c_1\right)\right)}.$$

We first want to demonstrate that

$$\frac{\partial}{\partial n_A}\frac{\overline{X}_{kj}^{I}}{X_{kj}^{FB}} = \frac{\overline{X}_{kj}^{F}\partial\overline{X}_{kj}^{I}/\partial n_A - \overline{X}_{kj}^{I}\partial\overline{X}_{kj}^{F}/\partial n_A}{\left(\overline{X}_{kj}^{F}\right)^2} \geq 0. \tag{10}$$

As long as $\partial\overline{X}_{kj}^{I}/\partial n_A \neq 0, \partial\overline{X}_{kj}^{F}/\partial n_A \neq 0$ (10) this is equivalent to

$$\frac{\overline{X}_{kj}^{F}}{\partial\overline{X}_{kj}^{F}/\partial n_A} - \frac{\overline{X}_{kj}^{I}}{\partial\overline{X}_{kj}^{I}/\partial n_A} \geq 0. \tag{11}$$

$$\frac{\overline{X}_{kj}^{F}}{\partial\overline{X}_{kj}^{F}/\partial n_A} - \frac{\overline{X}_{kj}^{I}}{\partial\overline{X}_{kj}^{I}/\partial n_A} = \frac{\Delta\left(-k(j+k) + \dfrac{c_1^2\left(n_A j - n_B k\right)^2}{\left(\left(n_B - j\right)c_1 + jc_2\right)\left(\left(n_B - j\right)c_1 + j\left(c_2 + \Delta\right)\right)}\right)}{jc_1} \tag{12}$$

We will study the shape of this expression as a function of $\Delta$. One can verify that

$$\begin{aligned}
\frac{\partial}{\partial\Delta}\left(\frac{\overline{X}_{kj}^{F}}{\partial\overline{X}_{kj}^{F}/\partial n_A} - \frac{\overline{X}_{kj}^{I}}{\partial\overline{X}_{kj}^{I}/\partial n_A}\right) &= \frac{-k(j+k) + \dfrac{c_1^2\left(n_A j - n_B k\right)^2}{\left(\left(n_B - j\right)c_1 + j\left(c_2 + \Delta\right)\right)^2}}{jc_1} \\
\frac{\partial^2}{\partial\Delta^2}\left(\frac{\overline{X}_{kj}^{F}}{\partial\overline{X}_{kj}^{F}/\partial n_A} - \frac{\overline{X}_{kj}^{I}}{\partial\overline{X}_{kj}^{I}/\partial n_A}\right) &= -\frac{2c_1\left(n_A j - n_B k\right)^2}{\left(\left(n_B - j\right)c_1 + j\left(c_2 + \Delta\right)\right)^3} \leq 0.
\end{aligned} \tag{13}$$

That is, $\left(\dfrac{\overline{X}_{kj}^{F}}{\partial\overline{X}_{kj}^{F}/\partial n_A} - \dfrac{\overline{X}_{kj}^{I}}{\partial\overline{X}_{kj}^{I}/\partial n_A}\right)$ is a concave function with a maximum in $\Delta$.

Because we are interested in showing that (11) is a positive function, we only need to check boundary cases Δ=0 and Δ=∞. Clearly, at Δ=0 (11) is zero because at Δ=0 First-Best and Second-Best solutions are equivalent.

Next we check Δ=∞. Recall that $\Delta = \frac{p(c_2 - c_1)}{1-p}$ so $\Delta \to \infty \Rightarrow p \to 1 \Rightarrow \{k \to n_A, j \to n_B\}$.

$\bar{X}^F_{n_A n_B} = \bar{X}^I_{n_A n_B} = \frac{Rn_A n_B}{2c_1(n_A + n_B)}$. Hence, $\left.\frac{\bar{X}^I_{kj}}{\bar{X}^F_{kj}}\right|_{\Delta=\infty} = 1$ and $\left.\frac{\partial}{\partial n_A}\frac{\bar{X}^I_{kj}}{\bar{X}^F_{kj}}\right|_{\Delta=\infty} \geq 0$.

To summarize, we have shown that for any problem parameters it is the case that $\frac{\partial}{\partial n_A}\frac{\bar{X}^I_{kj}}{\bar{X}^F_{kj}} \geq 0$.

Analogously, one can verify that $\frac{\partial}{\partial n_B}\frac{\bar{X}^I_{kj}}{\bar{X}^F_{kj}} \geq 0$. Hence, if $n_A = n_B = n$, we can vary $n_A$ and $n_B$ sequentially and Results 1 and 2 follow.

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